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A strain gradient Timoshenko beam element: application to MEMS. (English) Zbl 1323.74088

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Summary: The classical continuum theory not only underestimates the stiffness of microscale structures such as microbeams but is also unable to capture the size dependency, a phenomenon observed in these structures. Hence, the non-classical continuum theories such as the strain gradient elasticity have been developed. In this paper, a Timoshenko beam finite element is developed based on the strain gradient theory and employed to evaluate the mechanical behavior of microbeams used in microelectromechanical systems. The new beam element is a comprehensive beam element that recovers the formulations of strain gradient Euler-Bernoulli beam element, modified couple stress (another non-classical theory) Timoshenko and Euler-Bernoulli beam elements, and also classical Timoshenko and Euler-Bernoulli beam elements; note that the shear-locking phenomenon will not happen for the new Timoshenko beam element. The stiffness and mass matrices of the new element are derived in closed forms by following an energy-based approach and using Hamilton's principle. It is noted that unlike the classical beam elements, the stiffness matrix of the new element has a size-dependent nature that can capture the size-dependent behavior of microbeams. The shape functions of the newly developed beam element are determined by solving the equilibrium equations of strain gradient Timoshenko beams, which brings about a size-dependent characteristic for them. The new beam element is employed to evaluate the static deflection of a microcantilever, and the results are compared to the experimental data as well as the results obtained by using the classical beam element and the couple stress plane element. The new beam element is also implemented to calculate the static deflection, vibration frequency, and pull-in voltage of electrostatically actuated microbeams. The current results are compared to the experimental data as well as the classical FEM outcomes. It is observed that the results of the new element are in excellent agreement with the experimental data while the gap between the experimental and classical FEM results is significant.

MSC:

74S05 Finite element methods applied to problems in solid mechanics

74K10 Rods (beams, columns, shafts, arches, rings, etc.)

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

Full Text: DOI

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