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From global to local radial basis function collocation method for transport phenomena. en. (English) Zbl 1323.65111

Leitão, V. M. A. (ed.) et al., Advances in meshfree techniques. Invited contributions based on the presentation at the ECCOMAS thematic conference on meshless methods, Lisbon, Portugal, July 11–14, 2005. Dordrecht: Springer (ISBN 978-1-4020-6094-6/hbk; 978-90-481-7533-8/pbk; 978-1-4020-6095-3/ebook). Computational Methods in Applied Sciences (Springer) 5, 257-282 (2007).

Summary: This article introduces basic concepts of meshless methods for solving partial differential equations in their strong form by collocation or least squares approximation. Global and local formulations are defined. The current achievements, based on the local form and collocation with radial basis functions are explained in detail. Heat transfer and fluid flow problems are treated. These achievements represent a simple, and at the same time more efficient version of the classical meshless radial basis function collocation (Kansa) method. Instead of global, the collocation is made locally over a set of overlapping domains of influence and the time-stepping is performed in an explicit way. Only small systems of linear equations with the dimension of the number of nodes included in the domain of influence have to be solved for each node. The computational effort thus grows roughly linearly with the number of the nodes. The represented approach thus overcomes the principal large scale bottleneck of the original Kansa method and widely opens space for industrial applications of the method. The purpose of this article is to give a concentrated information on this new method, which has already been successfully applied in macroscopic and microscopic transport phenomena field, accompanied with research requirements for the future. It is devoted to practicing engineers and researchers.

For the entire collection see [Zbl 1125.74003].

MSC:

65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

76M25 Other numerical methods (fluid mechanics) (MSC2010)

76R99 Diffusion and convection

Keywords:

radial basis function collocation method; transport phenomena; strong formulation; multiquadrics

Full Text: DOI

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