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On the transverse scalar curvature of a compact Sasaki manifold. (English) Zbl 1323.53047
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Summary: We show that the standard picture regarding the notion of stability of constant scalar curvature metrics in Kähler geometry described by S. K. Donaldson, which involves the geometry of infinite dimensional groups and spaces, can be applied to the constant scalar curvature metrics in Sasaki geometry with only few modification. We prove that the space of Sasaki metrics is an infinite dimensional symmetric space and that the transverse scalar curvature of a Sasaki metric is a moment map of the strict contactomorphism group.

MSC:

53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)

Cited in **5** Documents

53C35 Differential geometry of symmetric spaces

58B25 Group structures and generalizations on infinite-dimensional manifolds

Keywords:

transverse scalar curvature; symmetric space; moment map

Full Text: [DOI](#) [arXiv](#)

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