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**Proof theory for locally finite many-valued logics: semi-projective logics.** (English)

Zbl 1322.03019

Theor. Comput. Sci. 480, 26-42 (2013).

As the authors explain, the aim of the article is to extend the methodology in [M. Baaz and C. G. Fermüller, Lect. Notes Comput. Sci. 1617, 36–50 (1999; Zbl 0931.03066)] to systematically construct analytic calculi for semi-projective logics – a large family of (propositional) locally finite many-valued logics. Such calculi, defined in the framework of sequents of relations, are proof search oriented and can be used to settle the computational complexity of the formalized logics. As a case study they derive sequent calculi of relations for nilpotent minimum logic (see [F. Esteva and L. Godo, Fuzzy Sets Syst. 124, No. 3, 271–288 (2001; Zbl 0994.03017)]) and for a family of axiomatic extensions of Hájek’s basic logic BL extended with the  $n$ -contraction axiom, (see [M. Bianchi and F. Montagna, Arch. Math. Logic 50, No. 3–4, 257–285 (2011; Zbl 1266.03041)]). The introduced calculi are used to prove that the decidability problem in these logics is Co-NP complete.

The main results are in Sections 3 to 5. Section 3 is about semi-projective logics, and presents a general method to construct an analytic calculi, for them. It is shown, among the other things, that every semi-projective logic with finitely-many connectives and constants is locally finite, and that all semi-projective logics are decidable. These results are then applied in Section 4, to present an analytic calculi for NM and for a family of  $n$ -contractive axiomatic extensions of BL. Moreover, it is also shown that such calculi provide a Co-NP decision procedure for the validity problem of these logics. Finally, in Section 5 is discussed the problem of defining analytic calculi for first-order many-valued logics, and most of the results are negative.

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#### MSC:

03B50 Many-valued logic

03F03 Proof theory in general (including proof-theoretic semantics)

03B25 Decidability of theories and sets of sentences

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#### Keywords:

many-valued logic; analytic calculi; sequents of relations; nilpotent minimum logic; basic logic; first-order logics; computational complexity

#### Software:

MUItlog

**Full Text:** DOI

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