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On the asymptotic behavior of a log gas in the bulk scaling limit in the presence of a varying external potential. I. (English) [Zbl 1321.82027](#)

Commun. Math. Phys. 337, No. 3, 1397-1463 (2015).

The authors study the determinant of the integral Fredholm's operator with kernel $\sin x(a-b)/\pi(a-b)$ which appears in the analysis of a log-gas of interacting particles. The main result is obtained in the form of a general expression for the logarithm of this determinant. The proof is lengthy and works via Riemann-Hilbert analysis.

Reviewer: [Guy Jumarie \(Montréal\)](#)

MSC:

[82C22](#) Interacting particle systems in time-dependent statistical mechanics
[15B52](#) Random matrices (algebraic aspects)
[15A18](#) Eigenvalues, singular values, and eigenvectors
[15A15](#) Determinants, permanents, traces, other special matrix functions
[60G55](#) Point processes (e.g., Poisson, Cox, Hawkes processes)

Cited in **1** Review
Cited in **15** Documents

Keywords:

[interacting particles](#); [Fredholm operator](#); [Riemann-Hilbert analysis](#); [scaling limit](#); [log-gas](#)

Software:

[DLMF](#)

Full Text: [DOI](#) [arXiv](#)

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