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How to produce S-tense operators on lattice effect algebras. (English) Zbl 1319.81013

Found. Phys. 44, No. 7, 792-811 (2014).

The paper continues the study of tense operators on MV-algebras, published in [*M. Botur* and *J. Paseka*, "On tense MV-algebras", Fuzzy Sets Syst. (2014); doi:10.1016/j.fss.2014.06.006] and [*J. Paseka*, Fuzzy Sets Syst. 232, 62–73 (2013; Zbl 1314.06016)] Here the results are extended to effect algebras.

Effect algebras offer a very general representation of quantum events, using a partial operation $+$ (disjunction of mutually excluding events) and orthosupplement (generalized negation) $'$. In lattice effect algebras, we may use total operation $x \oplus y = x + (y \wedge x')$ and its dual, $x \odot y = (x' \oplus y')'$.

Tense operators admit to introduce time and dynamics and express logical statements like "it is always going to be the case that ..." and "it has always been the case that ...". It is shown how tense operators can be constructed for a given time frame. The authors study also the inverse task, called a representation theorem: for given tense operators on an effect algebra, to find the respective time frame. The authors distinguish *E-tense operators*, which preserve $\oplus, \odot$ just as E-states, and *S-tense operators* G, which, moreover, are interior operators, i.e., $G(x) \leq x$, $G(G(x)) = x$.

A *state* is a finitely additive probability measure. If a state s on a lattice effect algebra satisfies $s(x \oplus x) = s(x) \oplus s(x)$, resp. $s(x) = 1 = s(y) \implies s(x \wedge y) = 1$, it is called an *E-state*, resp. a *Jauch-Piron state*. The E-states are extremal states, thus they do not form a convex set.

Further, *E-semi states* are studied; these are only subadditive and Jauch-Piron E-semi states are infima of sets of E-states.

If an S-tense effect algebra E has an order reflecting (=order determining) set of E-states and all E-states are Jauch-Piron states, then E can be embedded into a tense MV-algebra. A representation theorem for such lattice effect algebras with E-tense operators is proved and demonstrated on an example.

Reviewer: Mirko Navara (Praha)

MSC:

81P10	Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)	Cited in **2** Documents
06C15	Complemented lattices, orthocomplemented lattices and posets	
03G25	Other algebras related to logic	
06A11	Algebraic aspects of posets	

Keywords:

effect algebra; MV-algebra; complete lattice; tense operator; S-tense operator; Jauch-Piron E-state; Jauch-Piron E-semi-state

Full Text: DOI

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