**Hug, Daniel**; **Last, Günter**; **Pawlas, Zbynok**; **Weil, Wolfgang**
**Statistics for Poisson models of overlapping spheres.** (English) Zbl 1319.60014
Adv. Appl. Probab. 46, No. 4, 937-962 (2014).

Summary: In this paper we consider the stationary Poisson Boolean model with spherical grains and propose a family of nonparametric estimators for the radius distribution. These estimators are based on observed distances and radii, weighted in an appropriate way. They are ratio unbiased and asymptotically consistent for a growing observation window. We show that the asymptotic variance exists and is given by a fairly explicit integral expression. Asymptotic normality is established under a suitable integrability assumption on the weight function. We also provide a short discussion of related estimators as well as a simulation study.

**MSC:**

60D05 Geometric probability and stochastic geometry
60G57 Random measures
52A21 Convexity and finite-dimensional Banach spaces (including special norms, zonoids, etc.) (aspects of convex geometry)
60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
52A22 Random convex sets and integral geometry (aspects of convex geometry)
52A20 Convex sets in $n$ dimensions (including convex hypersurfaces)
53C65 Integral geometry
46B20 Geometry and structure of normed linear spaces
62G05 Nonparametric estimation



**Keywords:**

stochastic geometry; spatial statistic; contact distribution function; Boolean model; spherical typical grain; point process; nonparametric estimation; radius distribution; asymptotic normality

**Full Text:** DOI Euclid

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