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Asymptotics of a cubic sine kernel determinant. (English) Zbl 1318.82015

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Summary: The one-parameter family of Fredholm determinants $\det(I - \gamma K_{\text{csin}})$, $\gamma \in \mathbb{R}$, is studied for an integrable Fredholm operator K_{csin} that acts on the interval $(-s, s)$ and whose kernel is a cubic generalization of the sine kernel that appears in random matrix theory. This Fredholm determinant arises in the description of the Fermi distribution of semiclassical nonequilibrium Fermi states in condensed matter physics as well as in the random matrix theory. By using the Riemann-Hilbert method, the large s asymptotics of $\det(I - \gamma K_{\text{csin}})$ is calculated for all values of the real parameter γ .

MSC:

- [82B23](#) Exactly solvable models; Bethe ansatz
- [33E05](#) Elliptic functions and integrals
- [34E05](#) Asymptotic expansions of solutions to ordinary differential equations
- [34M50](#) Inverse problems (Riemann-Hilbert, inverse differential Galois, etc.) for ordinary differential equations in the complex domain
- [82C23](#) Exactly solvable dynamic models in time-dependent statistical mechanics
- [15B52](#) Random matrices (algebraic aspects)
- [15A15](#) Determinants, permanents, traces, other special matrix functions

Cited in 1 Document

Keywords:

Fredholm determinant; integrable Fredholm operator; Riemann-Hilbert method; Fermi distribution

Full Text: [DOI](#) [arXiv](#)

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