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Application of the principle of minimizing the derivative to the construction of finite-difference schemes for computing discontinuous solutions of gas dynamics. (Reprint). (English. Russian original) Zbl 1316.76063

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Summary: Progress in computer technology now allows the application of numerical methods to some practical multidimensional steady and unsteady gas flows. While selecting a numerical method, preference should be given to higher-order schemes, since they allow the study of the fine features of the solution and the reduction of the computation time. In general, however, solutions of non-stationary nonlinear equations of gas dynamics are not smooth and may include strong discontinuities, such as shock waves or contact discontinuities. This fact is important for choosing an appropriate numerical scheme. If the strong discontinuities are extracted from the simulation domain and traced (shock fitting), while the rest of the flow is smooth, it is possible to use numerical schemes with approximation close to the second order. However, if the number of discontinuities increases with time leading to a complicated pattern, the application of the shock-fitting approach becomes difficult. In this case it is preferable to apply shock-capturing schemes, which have a lower order of approximation but allow the integration of the governing equations through the discontinuities without extracting their surfaces. Shock-capturing schemes of a higher order of approximation are currently under development.

In this paper, certain aspects of constructing a numerical scheme applicable to discontinuous flows of gas dynamics are discussed using the idea of minimizing the derivatives of the solution. A numerical scheme that is second-order accurate in space and first-order accurate in time is developed for the model equation $u_t + u_x = 0$. The properties of this scheme are analyzed and the conditions for its stability and monotonicity are formulated. These results are then used in devising a similar scheme for the non-stationary one-dimensional equations of gas dynamics. The performance of the new scheme is compared to that of other schemes, using the break-up of an initial discontinuity (Riemann problem) as an example.

MSC:

76M20 Finite difference methods applied to problems in fluid mechanics
76N15 Gas dynamics (general theory)

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