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Modelling a wolbachia invasion using a slow-fast dispersal reaction-diffusion approach. (English) Zbl 1311.92173

Bull. Math. Biol. 75, No. 9, 1501-1523 (2013).

Summary: This paper uses a reaction-diffusion approach to examine the dynamics in the spread of a *Wolbachia* infection within a population of mosquitoes in a homogeneous environment. The formulated model builds upon an earlier model by *G. T. Skalski* and *J. F. Gilliam* ["A diffusion-based theory of organism dispersal in heterogeneous populations", Am. Nat. 161, No. 3, 441–458 (2003; doi:10.1086/367592)], which incorporates a slow and fast dispersal mode. This generates a faster wavespeed than previous reaction-diffusion approaches, which have been found to produce wavespeeds that are unrealistically slow when compared with direct observations. In addition, the model incorporates cytoplasmic incompatibility between male and female mosquitoes, which creates a strong Allee effect in the dynamics. In previous studies, linearised wavespeeds have been found to be inaccurate when a strong Allee effect is underpinning the dynamics. We provide a means to approximate the wavespeed generated by the model and show that it is in close agreement with numerical simulations. Wavespeeds are approximated for both *Aedes aegypti* and *Drosophila simulans* mosquitoes at different temperatures. These wavespeeds indicate that as the temperature decreases within the optimal temperature range for mosquito survival, the speed of a *Wolbachia* invasion increases for *Aedes aegypti* populations and decreases for *Drosophila simulans* populations.

MSC:

92D30	Epidemiology
35K57	Reaction-diffusion equations

Keywords:
invasion dynamics; Wolbachia invasion; reaction-diffusion equations; strong Allee effect

Full Text: DOI

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