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**Queues and risk models with simultaneous arrivals.** (English) Zbl 1311.60103

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Consider a multidimensional classical risk model. That is, claims occur according to a Poisson process, and the claim sizes are i.i.d. vectors independent of the claim arrivals. The premiums are earned linearly in time. By changing the monetary units, one can assume that the premium rate is one in each of the coordinates.

Using the arguments in [*S. Asmussen* and *S. S. Petersen*, *Adv. Appl. Probab.* 20, No. 4, 913–916 (1988; [Zbl 0657.60111](#))], a dual version of a multivariate queueing model is obtained. In this way, ruin probabilities can be expressed as the stationary distribution in the queueing model. The Laplace transform of the stationary distribution is found. Using Rouché's theorem, a unique zero of the characteristic equation can be identified. This zero has an interpretation connected to the distribution of the additional workload if the shortest queue becomes idle.

It is further assumed that at each arrival the service times are ordered. Thus, the first queue always gets the largest service time, the last queue the shortest. Considering an insurance surplus, this limits the applicability of the results in risk theory. In the queueing context the last queue gets idle first. The stationary workload can then be decomposed. The workload is the sum of independent workloads, where in the second sum the last server is idle, in the third sum, the last but one server is idle, etc. In the last sum, all servers but the first are idle. These sums have the following interpretations. Consider a queue where all servers empty at the time the last server becomes idle. The first sum is then the stationary distribution of this modified queue. The sum of the first two components gives the stationary distribution if the whole queue empties when the last but one server becomes idle. In the case of two servers, more explicit results are given.

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#### MSC:

- [60K25](#) Queueing theory (aspects of probability theory)
- [60J25](#) Continuous-time Markov processes on general state spaces
- [90B22](#) Queues and service in operations research
- [91B30](#) Risk theory, insurance (MSC2010)

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#### Keywords:

[queues with simultaneous arrival](#); [stationary distribution](#); [stochastic decomposition](#); [duality](#); [multivariate risk model](#)

**Full Text:** [DOI](#) [Euclid](#) [arXiv](#)

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