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Degrees of categoricity and the hyperarithmetic hierarchy. (English) Zbl 1311.03070

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Let **d** be a Turing degree. A computable structure is **d**-*computably categorical* if any of its computable isomorphic copies is isomorphic to it via a **d**-computable isomorphism. If there is a least degree with this property, then this degree is called the *degree of categoricity* of this structure. A degree is called a degree of categoricity if it is the degree of categoricity for some computable structure. If **d** is a degree of categoricity with the property that there are isomorphic computable structures $\mathcal{A}_0$ and $\mathcal{A}_1$ for which **d** is the degree of categoricity and every isomorphism from $\mathcal{A}_0$ onto $\mathcal{A}_1$ computes **d**, then **d** is called *strong degree of categoricity.*

The authors prove the following results:

1) for any computable ordinal α, $\mathbf{0}^{(\alpha)}$ is the strong degree of categoricity for some computable structure;

2) if in addition α is a successor ordinal, then any degree 2-c.e. in and above $\mathbf{0}^{(\alpha)}$ is a strong degree of categoricity;

3) every degree of categoricity is hyperarithmetic;

4) the set of codes of all structures having a degree of categoricity is Π^1_1-complete.

Reviewer: Andrei S. Morozov (Novosibirsk)

MSC:

03D45 Theory of numerations, effectively presented structures
03D28 Other Turing degree structures

Cited in **40** Documents

Keywords:

computability theory; computable structure theory; Turing degrees; isomorphisms; relative categoricity; degree of categoricity

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