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**A survey of fuzzy implication algebras and their axiomatization.** (English) Zbl 1308.03063  
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The paper reviews the main results concerning FI-algebras. Basic concepts and main properties of FI-algebras and RFI-algebras are discussed in Section 2, where, in addition, the relationships between FI-algebras and BCK-algebras are analyzed. The relationships between RFI-algebras and residuated lattices, MTL-algebras, BL-algebras, MV-algebras, G-algebras and  $\Pi$ -algebras are discussed in Section 3. Section 4 reviews the relationships between CFI-algebras and MV-algebras. An important property is that the classes of all MTL-algebras, BL-algebras and MV-algebras are subclasses of the class of all FI-algebras. The concepts of MP-filter, P-filter, Q-filter and C-filter are recalled in the last part of this section. Section 5 discusses main properties of HFI-algebras, another subclass of FI-algebras. Two other subclasses of FI-algebras, called PFI-algebras and  $W_d$ -FI-algebras, are analyzed in Section 6. A formal system for formalizing FI-algebras is given in Section 7. This system is extended to formalize some important subclasses of FI-algebras: RFI-algebras, CFI-algebras and HFI-algebras. Finally, some semantic extensions of the MTL formal system are pointed out.

Reviewer: **Ion Iancu (Craiova)**

**MSC:**

**03G25** Other algebras related to logic

**06D35** MV-algebras

**06F35** BCK-algebras, BCI-algebras

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