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Asymptotic properties of polynomials orthogonal with respect to varying weights, and related topics of spectral theory. (English. Russian original) Zbl 1304.37054

St. Petersbg. Math. J. 25, No. 2, 223-240 (2014); translation from Algebra Anal. 25, No. 2, 101-124 (2013).

The paper deals with the asymptotic properties as $n \to \infty$ of the coefficients of the three-term relations

$$a_{l-1}^{(n)} p_{l-1}^{(n)}(x) + b_l^{(n)} p_l^{(n)}(x) + a_l^{(n)} p_{l+1}^{(n)}(x) = \lambda p_l^{(n)}(x)$$

for polynomials orthogonal with respect to a varying weight $e^{-nV(x)}$, where V is a continuous function such that $V(x) \geq (1+\varepsilon)\log(1+x^2)$. The support of the corresponding equilibrium measure in this case consists of a finite number g of finite intervals. The asymptotic formulas for the coefficients $a_{n-1}^{(n)}$ and $b_{n-1}^{(n)}$ were obtained by Deift, Kriecherbauer, McLaughlin and Zhou via a Riemann-Hilbert problem approach. Using the same approach, the authors establish the asymptotics of the coefficients $a_{n+k}^{(n)}$ and $b_{n+k}^{(n)}$ as $n \to \infty$ for $k \in \mathbb{Z}$. The following property of these coefficients is obtained: Let $\mathbf{x} \in [0,1]^g$ be an arbitrary vector; then, there exists a subsequence $n_i(\mathbf{x}) \to \infty$ such that

$$a_{n_i(\mathbf{x})+k}^{(n_i(\mathbf{x}))} \to a_k(\mathbf{x}), \quad b_{n_i(\mathbf{x})+k}^{(n_i(\mathbf{x}))} \to b_k(\mathbf{x}),$$

where $\{a_k(\mathbf{x}), b_k(\mathbf{x})\}$ are the coefficients of a finite gap Jacobi operator. This operator has the continuous spectrum on the support of an equilibrium measure. The formula for the initial divisor depending on $\mathbf{x}$ is given.

Reviewer: Leonid Golinskii (Kharkov)

MSC:

37K40 Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems
35Q53 KdV equations (Korteweg-de Vries equations)
37K45 Stability problems for infinite-dimensional Hamiltonian and Lagrangian systems
35Q15 Riemann-Hilbert problems in context of PDEs

Keywords:

Riemann-Hilbert problem; asymptotics; varying weights; spectral theory

Full Text: DOI

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