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Discontinuity waves as tipping points: applications to biological & sociological systems.

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Summary: The 'tipping point' phenomenon is discussed as a mathematical object, and related to the behaviour of nonlinear discontinuity waves in the dynamics of topical sociological and biological problems. The theory of such waves is applied to two illustrative systems in particular: a crowd-continuum model of pedestrian (or traffic) flow; and an hyperbolic reaction-diffusion model for the spread of the hantavirus infection (a disease carried by rodents). In the former, we analyse propagating acceleration waves, demonstrating how blow-up of the wave amplitude might indicate formation of a 'human-shock', that is, a 'tipping point' transition between safe pedestrian flow, and a state of overcrowding. While in the latter, we examine how travelling waves (of both acceleration and shock type) can be used to describe the advance of a hantavirus infection-front. Results from our investigation of crowd models also apply to equivalent descriptions of traffic flow, a context in which acceleration wave blow-up can be interpreted as emergence of the 'phantom congestion' phenomenon, and 'stop-start' traffic motion obeys a form of wave propagation.

MSC:

35L60 First-order nonlinear hyperbolic equations
35K57 Reaction-diffusion equations
35L67 Shocks and singularities for hyperbolic equations
92D30 Epidemiology
90B20 Traffic problems in operations research

Keywords:

traffic modelling; crowd dynamics; hantavirus; SIS epidemic model; hyperbolic reaction-diffusion equations

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References:

[1] G. Abramson, Spatiotemporal patterns in the Hantavirus infection,, Physical Review E, 66, (2002)· doi:10.1103/PhysRevE.66.011912

[2] L. J. S. Allen, Mathematical models for hantavirus infection in rodents,, Bulletin of Mathematical Biology, 68, 511, (2006) · Zbl 1334.92387 · doi:10.1007/s11538-005-9034-4

[3] L. J. S. Allen, A habitat-based model for the spread of hantavirus between reservoir and spillover species,, Journal of Theoretical Biology, 260, 510, (2009)· doi:10.1016/j.jtbi.2009.07.009

[4] A. Aw, Resurrection of "Second Order" models of traffic flow,, SIAM Journal on Applied Mathematics, 60, 916, (2000) · Zbl 0957.35086 · doi:10.1137/S0036139997332099

[5] E. Barbera, A hyperbolic reaction diffusion model for the hantavirus infection,, Mathematical Methods in the Applied Sciences, 31, 481, (2008) · Zbl 1180.35340 · doi:10.1002/mma.929

[6] A. D. Barnosky, Approaching a state shift in Earth's biosphere,, Nature, 486, 52, (2012)· doi:10.1038/nature11018

[7] N. Bellomo, On the modelling crowd dynamics from scaling to hyperbolic macroscopic models,, Math. Models Methods Appl. Sci., 18, 1317, (2008) · Zbl 1198.92036 · doi:10.1142/S0218202508003054

[8] N. Bellomo, On the modeling of traffic and crowds: A survey of models, speculations, and perspectives,, SIAM Review, 53, 409, (2011) · Zbl 1231.90123 · doi:10.1137/090746677

[9] N. Bellomo, On the modeling of crowd dynamics: Looking at the beautiful shapes of swarms,, Networks And Heterogeneous Media, 6, 383, (2011) · Zbl 1260.90052 · doi:10.3934/nhm.2011.6.383

[10] N. Bellomo, Modeling crowd dynamics from a complex system viewpoint,, Math. Models Methods Appl. Sci., 22, (2012) · Zbl 1252.35192 · doi:10.1142/S0218202512300049

[11] R. A. Bentley, Tipping points, animal culture, and social learning,, Current Zoology, 58, 298, (2012)

[12] R. A. Bentley, Cultural evolutionary tipping points in the storage and transmission of information,, Frontiers in Psychology, 3, (2013)· doi:10.3389/fpsyg.2012.00569

[13] P. J. Chen, Growth and decay of waves in solids„ in Handbuch der Physik, VIa/3, 303, (1973)

[14] C. I. Christov, Heat conduction paradox involving second sound propagation in moving media„ Physical Review Letters, 94, (2005)· doi:10.1103/PhysRevLett.94.154301

[15] I. Christov, On the propagation of second sound in nonlinear media: Shock, acceleration and travelling wave results„ J. Thermal Stresses, 33, 1109, (2010)· doi:10.1080/01495739.2010.517674

[16] I. Christov, Nonlinear acoustic propagation in homentropic perfect gases: A numerical study„ Physics Letters A, 353, 273, (2006)· doi:10.1016/j.physleta.2005.12.101

[17] I. Christov, Modelling weakly nonlinear acoustic wave propagation„ Quart. Jl. Mech. Appl. Math., 60, 473, (2007) · Zbl 1125.76379 · doi:10.1093/qjmam/hbm017

[18] M. Ciarletta, Poroacoustic acceleration waves„ Proceedings of the Royal Society A, 462, 3493, (2006) · Zbl 1149.74345 · doi:10.1098/rspa.2006.1730

[19] M. Ciarletta, Thermo-poroacoustic acceleration waves in elastic materials with voids without energy dissipation„ Int. J. Engng. Sci., 45, 736, (2007) · Zbl 1213.74095 · doi:10.1016/j.ijengsci.2007.05.001

[20] M. Ciarletta, Poroacoustic acceleration waves in a Jordan-Darcy-Cattaneo material„ Int. J. Non-linear Mech., 52, 8, (2013)· doi:10.1016/j.ijnonlinmec.2013.01.020

[21] C. M. Dafermos, Hyperbolic Conservation Laws in Continuum Physics„ volume 325 of Grundleheren der mathematischen Wissenschaften, (2010) · Zbl 1196.35001 · doi:10.1007/978-3-642-04048-1

[22] C. M. Dafermos, Development of singularities in solutions of the equations of nonlinear thermoelasticity„ Quart. Appl. Math., 44, 463, (1986) · Zbl 0661.35009

[23] J. W. Eslick, A dynamical study of the evolution of pressure waves propagating through a semi-infinite region of homogeneous gas combustion subject to a time-harmonic signal at the boundary„ Int. J. Non-linear Mech., 47, 18, (2012)· doi:10.1016/j.ijnonlinmec.2011.11.007

[24] M. Fabrizio, Electromagnetism of Continuous Media„ Oxford University Press, (2003) · Zbl 1027.78001 · doi:10.1093/acprof:oso/9780198527008.001.0001

[25] J. A. Foley, Tipping Points in the Tundra„ Science, 310, 627, (2005)

[26] Y. B. Fu, The transistion from acceleration wave to shock wave„ Int. J. Engng. Sci., 29, 617, (1991) · Zbl 0734.73013 · doi:10.1016/0020-7225(91)90066-C

[27] T. Gultop, Propagation of acceleration waves in the johnson-segalman fluids„ Mech. Res. Communications, 37, 153, (2010) · Zbl 1272.76026 · doi:10.1016/j.mechrescom.2009.12.007

[28] M. W. Hirsch, Differential Equations, Dynamical Systems, and Linear Algebra„ Academic Press, (1974) · Zbl 0309.34001

[29] R. L. Hughes, A continuum theory for the flow of pedestrians„ Transportation Research Part B, 36, 507· doi:10.1016/S0191-2615(01)00015-7

[30] D. Iesan, Thermoelastic Deformations„ Kluwer, (1996) · Zbl 0905.73001 · doi:10.1007/978-94-017-3517-9

[31] C. B. Jonsson, A global perspective on hantavirus ecology, epidemiology, and disease„ Clinical Micribiology Reviews, 23, 412, (2010)· doi:10.1128/CMR.00062-09

[32] P. M. Jordan, Growth and decay of shock and acceleration waves in a traffic flow model with relaxation„ Physica D, 207, 220, (2005) · Zbl 1078.35073 · doi:10.1016/j.physd.2005.06.002

[33] P. M. Jordan, Growth, decay and bifurcation of shock amplitudes under the type-II flux law„ Proc. Roy. Soc. London A, 463, 2783, (2007) · Zbl 1132.35413 · doi:10.1098/rspa.2007.1895

[34] P. M. Jordan, Some remarks on nonlinear poroacoustic phenomena„ Math. Computers Simulation, 80, 202, (2009) · Zbl 1422.76173 · doi:10.1016/j.matcom.2009.06.004

[35] P. M. Jordan, A note on Chrystal's equation„ Appl. Math. Computation, 217, 933, (2010) · Zbl 1298.34002 · doi:10.1016/j.amc.2010.05.095

[36] P. M. Jordan, A note on poroacoustic travelling waves under Darcy's law: Exact solutions„ Applications of Mathematics, 56, 99, (2011) · Zbl 1224.33014 · doi:10.1007/s10492-011-0011-6

[37] P. M. Jordan, On the propagation of nonlinear acoustic waves in viscous and thermoviscous fluids„ European J. Mech., 34, 56, (2012) · Zbl 1258.76143 · doi:10.1016/j.euromechflu.2012.01.016

[38] P. M. Jordan, Compact acoustic travelling waves in a class of fluids with nonlinear material dispersion„ Proc. Roy. Soc. London A, 468, 3441, (2012) · Zbl 1371.76118 · doi:10.1098/rspa.2012.0321

[39] S. Kefi, Spatial vegetation patterns and imminent desertification in Mediterranean arid ecosystems„ Nature, 449, 213, (2007)· doi:10.1038/nature06111

[40] K. A. Lindsay, Acceleration Waves and Second Sound in a Perfect fluid„ Archive for Rational Mechanics and Analysis, 68, 53, (1978) · Zbl 0399.76078 · doi:10.1007/BF00276179

[41] R. S. Mani, Emerging Viral Infections in India„ Proceedings of the National Academy of Sciences, 82, 5, (2012)· doi:10.1007/s40011-011-0001-1

[42] A. Marasco, On the first-order speeds in any direction of acceleration waves in prestressed second - order isotropic, compressible, and homogeneous materials„ Mathematical and Computer Modelling, 49, 1644, (2009) · Zbl 1165.74305 · doi:10.1016/j.mcm.2008.07.037

[43] A. Marasco, Second - order effects on the wave propagation in elastic, isotropic, incompressible, and homogeneous media„ Int. J. Engng. Sci., 47, 499, (2009) · Zbl 1213.35384 · doi:10.1016/j.ijengsci.2008.08.009

[44] A. Marasco, On the acceleration waves in second - order elastic, isotropic, compressible, and homogeneous materials„ Mathematical and Computer Modelling, 49, 1504, (2009) · Zbl 1165.74333 · doi:10.1016/j.mcm.2008.06.005

[45] V. Méndez, Dynamics and thermodynamics of delayed population growth„ Physical Review E, 55, 6476, (1997)

[46] V. Méndez, Speed of wave-front solutions to hyperbolic reaction-diffusion equations„ Physical Review E, 60, 5231, (1999)· doi:10.1103/PhysRevE.60.5231

[47] J. N. Mills, Long-Term studies of hantavirus reservoir populations in the southwestern united states: Rationale, potential, and methods„ Emerging Infectious Diseases, 5, 95, (1999)· doi:10.3201/eid0501.990111

[48] A. Miranville, A phase-field model Based on a three-phase-lag heat conduction„ Applied Mathematics and Optimization, 63, 133, (2011) · Zbl 1213.35111 · doi:10.1007/s00245-010-9114-9

[49] A. Miranville, On a phase-field model with a logarithmic nonlinearity„ Applications of Mathematics, 57, 215, (2012) · Zbl 1265.35139 · doi:10.1007/s10492-012-0014-y

[50] M. Ostoja-Starzewski, Stochastic dynamics of acceleration waves in random media„ Mechanics of Materials, 38, 840, (2006) · Zbl 1149.76669 · doi:10.1016/j.mechmat.2005.06.022

[51] P. Paoletti, Acceleration waves in complex materials„ Discrete Continuous Dyn. Systems B, 17, 637, (2012) · Zbl 1451.74124 · doi:10.3934/dcdsb.2012.17.637

[52] M. Rietkerk, Self-Organized patchiness and catastrophic shifts in ecosystems„ Science, 305, 1926, (2004)· doi:10.1126/science.1101867

[53] F. Sauvage, Modelling hantavirus in fluctuating populations of bank voles: The role of indirect transmission on virus persistence„ Journal of Animal Ecology, 72, 1, (2003)· doi:10.1046/j.1365-2656.2003.00675.x

[54] M. Scheffer, Early-warning signals for critical transitions„ Nature, 461, 53, (2009)· doi:10.1038/nature08227

[55] M. Scheffer, Anticipating critical transitions„ Science, 338, 344, (2012)· doi:10.1126/science.1225244

[56] C. Schmaljohn, Hantaviruses: A global disease problem„ Emerging Infectious Diseases, 3, 95, (1997)· doi:10.3201/eid0302.970202

[57] V. D. Sharma, Evolution of weak shocks in one dimensional planar and non-planar gasdynamic flows„ Int. J. Non-linear Mech., 47, 918, (2012)· doi:10.1016/j.ijnonlinmec.2012.06.001

[58] F. M. F. Simoes, Instabilities in elastic-plastic fluid-saturated porous media: Harmonic wave versus acceleration wave analyses„ Int. J. Solids Structures, 36, 1277, (1999) · Zbl 0953.74018 · doi:10.1016/S0020-7683(98)00002-X

[59] B. Straughan, Stability, and Wave Motion in Porous Media„ volume 165 of Appl. Math. Sci., (2008) · Zbl 1149.76002

[60] B. Straughan, Heat Waves„ volume 177 of Appl. Math. Sci., (2011) · Zbl 1232.80001 · doi:10.1007/978-1-4614-0493-4

[61] B. Straughan, Tipping points in Cattaneo-Christov thermohaline convection„ Proc. Roy. Soc. London A, 467, 7, (2011) · Zbl 1219.76049 · doi:10.1098/rspa.2010.0104

[62] B. Straughan, Thermo-poroacoustic acceleration waves in elastic materials with voids„ in Encyclopedia of thermal stresses, (2013) · Zbl 1185.74024

[63] Y. Sugiyama, Traffic jams without bottlenecks-experimental evidence for the physical mechanism of the formation of a jam„ New Journal of Physics, 10, (2008)· doi:10.1088/1367-2630/10/3/033001

[64] C. Truesdell, The Classical Field Theories„ article in Handbuch der Physik, (1960) · Zbl 0096.00301

[65] C. A. Truesdel, The Non-Linear Field Theories of Mechanics„ Springer, (1992)· doi:10.1115/1.3625229

[66] G. Walker, The tipping point of the iceberg„ Nature, 441, 802, (2006)

[67] D. H. Wall, Global change tipping points: above- and below-ground biotic interactions in a low diversity ecosystem„ Philosophical Transactions of the Royal Society B, 362, 2291, (2007)· doi:10.1098/rstb.2006.1950

[68] G. B. Whitham, Linear and Non-Linear Waves„ Wiley, (1974) · Zbl 0373.76001

[69] H. M. Zhang, A non-equilibrium traffic model devoid of gas-like behaviour„ Transportation Research Pat B, 36, 275, (2002)· doi:10.1016/S0191-2615(00)00050-3