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Spatially homogeneous solutions of the Vlasov-Nordström-Fokker-Planck system. (English)

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Summary: The Vlasov-Nordström-Fokker-Planck system describes the evolution of self-gravitating matter experiencing collisions with a fixed background of particles in the framework of a relativistic scalar theory of gravitation. We study the spatially-homogeneous system and prove global existence and uniqueness of solutions for the corresponding initial value problem in three momentum dimensions. Additionally, we study the long time asymptotic behavior of the system and prove that even in the absence of friction, solutions possess a non-trivial asymptotic profile. An exact formula for the long time limit of the particle density is derived in the ultra-relativistic case.

MSC:

35Q84	Fokker-Planck equations
82C31	Stochastic methods (Fokker-Planck, Langevin, etc.) applied to problems in time-dependent statistical mechanics
35Q83	Vlasov equations
35B40	Asymptotic behavior of solutions to PDEs
35A01	Existence problems for PDEs: global existence, local existence, non-existence
35A02	Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness

Keywords:

Vlasov-Nordström; Fokker-Planck equation; spatially homogeneous; global existence; ultra-relativistic; long time behavior

Full Text: DOI arXiv

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