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Quantifying Bayesian filter performance for turbulent dynamical systems through information theory. (English) Zbl 1302.93212

Summary: Incomplete knowledge of the true dynamics and its partial observability pose a notoriously difficult problem in many scientific applications which require predictions of high-dimensional dynamical systems with instabilities and energy fluxes across a wide range of scales. In such cases assimilation of real data into the modeled dynamics is necessary for mitigating model error and for improving the stability and predictive skill of imperfect models. However, the practically implementable data assimilation/filtering strategies are also imperfect and not optimal due to the formidable complexity nature of the underlying dynamics. Here, the connections between information theory and the filtering problem are exploited in order to establish bounds on the filter error statistics, and to systematically study the statistical accuracy of various Kalman filters with model error for estimating the dynamics of spatially extended, partially observed turbulent systems. The effects of model error on filter stability and accuracy in this high-dimensional setting are analyzed through appropriate information measures which naturally extend the common path-wise estimates of filter performance, like the mean-square error or pattern correlation, to the statistical superensemble setting that involves all possible initial conditions and all realizations of noisy observations of the truth signal. Particular emphasis is on the notion of practically achievable filter skill which requires trade-offs between different facets of filter performance; a new information criterion is introduced in this context. This information-theoretic framework for assessment of filter performance has natural generalizations to Kalman filtering with non-Gaussian statistically exactly solvable forecast models. Here, this approach is utilized to study the performance of imperfect, reduced-order filters involving Gaussian forecast models which use various spatio-temporal discretizations to approximate the dynamics of the stochastically forced advection-diffusion equation; important examples in this configuration include effects of biases due to model error in the filter estimates for the mean dynamics which are quantified through appropriate information measures.

MSC:
93E11 Filtering in stochastic control theory
62B10 Statistical aspects of information-theoretic topics
94A17 Measures of information, entropy
60G35 Signal detection and filtering (aspects of stochastic processes)
60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)
93E10 Estimation and detection in stochastic control theory
60H30 Applications of stochastic analysis (to PDEs, etc.)
35C20 Asymptotic expansions of solutions to PDEs
76F55 Statistical turbulence modeling

Keywords:
Kalman filter; information theory; uncertainty quantification; turbulent dynamical systems

Full Text: DOI