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Proper and adjoint exhausters in nonsmooth analysis: optimality conditions. (English)

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Summary: The notions of upper and lower exhausters represent generalizations of the notions of exhaustive families of upper convex and lower concave approximations (u.c.a., l.c.a.). The notions of u.c.a.'s and l.c.a.'s were introduced by *B. N. Pshenichnyi* [Convex analysis and extremal problems. Seriya "Nelineĭnyĭ Analiz i ego Prilozheniya". Moskva: "Nauka" (1980; Zbl 0477.90034)], while the notions of exhaustive families of u.c.a.'s and l.c.a.'s were described by *V. F. Demyanov* and *A. M. Rubinov* ["Elements of quasidifferential calculus". In: Nonsmooth Problems of Optimization Theory and Control, Leningrad University Press, Leningrad, 5–127 (1982)]. These notions allow one to solve the problem of optimization of an arbitrary function by means of Convex Analysis thus essentially extending the area of application of Convex Analysis. In terms of exhausters it is possible to describe extremality conditions, and it turns out that conditions for a minimum are expressed via an upper exhauster while conditions for a maximum are formulated in terms of a lower exhauster (see, e.g. [*V. F. Demyanov* and *V. A. Roshchina*, Appl. Comput. Math. 4, No. 2, 114–124 (2005; Zbl 1209.90349); Optimization 55, No. 5–6, 525–540 (2006; Zbl 1156.90458)]). This is why an upper exhauster is called a proper exhauster for minimization problems while a lower exhauster is called a proper one for maximization problems. The results obtained provide a simple geometric interpretation and allow one to construct steepest descent and ascent directions. Until recently, the problem of expressing extremality conditions in terms of adjoint exhausters remained open. Demyanov and Roshchina [2005, 2007, loc. cit.] was the first to derive such conditions. However, using the conditions obtained (unlike the conditions expressed in terms of proper exhausters) it was not possible to find directions of descent and ascent. In [*M. E. Abbasov*, "Extremality conditions in terms of adjoint exhausters" (Russian), Vestnik of Saint-Petersburg University; Ser. 10. Applied mathematics, informatics, control processes. N. 2., 3–8. (2011)] new extremality conditions in terms of adjoint exhausters were discovered. In the present paper, a different proof of these conditions is given and it is shown how to find steepest descent and ascent conditions in terms of adjoint exhausters. The results obtained open the way to constructing numerical methods based on the usage of adjoint exhausters thus avoiding the necessity of converting the adjoint exhauster into a proper one.

MSC:

90C30 Nonlinear programming

90C46 Optimality conditions and duality in mathematical programming

Cited in 13 Documents

Keywords:

nonsmooth analysis; proper and adjoint exhausters and coexhausters; extremality conditions; converters

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