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On the identifiability of overcomplete dictionaries via the minimisation principle underlying K-SVD.  (English) [Zbl 1297.94018]

Summary: This article gives theoretical insights into the performance of K-SVD, a dictionary learning algorithm that has gained significant popularity in practical applications. The particular question studied here is when a dictionary $\Phi \in \mathbb{R}^{d \times K}$ can be recovered as local minimum of the minimisation criterion underlying K-SVD from a set of $N$ training signals $y_n = \Phi x_n$. A theoretical analysis of the problem leads to two types of identifiability results assuming the training signals are generated from a tight frame with coefficients drawn from a random symmetric distribution. First, asymptotic results showing that in expectation the generating dictionary can be recovered exactly as a local minimum of the K-SVD criterion if the coefficient distribution exhibits sufficient decay. Second, based on the asymptotic results it is demonstrated that given a finite number of training samples $N$, such that $N/\log N = O(K^3d)$, except with probability $O(N^{-Kd})$ there is a local minimum of the K-SVD criterion within distance $O(KN^{-1/4})$ to the generating dictionary.

MSC:
94A20 Sampling theory in information and communication theory
15A23 Factorization of matrices
68T05 Learning and adaptive systems in artificial intelligence
94A29 Source coding

Keywords:
dictionary learning; sparse coding; sparse component analysis; K-SVD; finite sample size; sampling complexity; dictionary identification; minimisation criterion; sparse representation

Software:
CoSaMP; PDCO

Full Text: DOI arXiv

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