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**A multiscale approach for modeling progressive damage of composite materials using fast Fourier transforms.** (English) [Zbl 1295.74006](#)  
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Summary: Composite materials possess a highly complex material behavior, and thus advanced simulation techniques are necessary to compute their mechanical response. In this regard, especially modeling failure and progressive damage presents a challenging task. Conventional macro mechanical methods and even closed form estimates are in many cases not sufficient to predict the appropriate mechanical material response. Full-field simulations must be resorted to, but these are known to be very expensive from the computational point of view. In this contribution we propose a more efficient multiscale approach similar to  $FE^2$ . Nonlinear material effects caused by progressive damage behavior are captured directly on the discretized material level using simple isotropic continuum damage laws. In contrast to conventional  $FE^2$  methods which use the Finite Element Method (FEM) to solve both scales numerically, the fine scale problem (material level) is rewritten in an integral form of Lippmann-Schwinger type and solved efficiently using the fast Fourier transformation (FFT). The calculation is carried out on a regular voxel grid that can be obtained from 3D images like tomographies. The fine scale problem is integrated in a standard Finite Element framework which is used to solve the macroscopic BVP (component level). In the work at hand, the scale coupling technique and the computation of the macroscopic tangent are described, and in some numerical examples the convergence behavior of the macroscopic Newton algorithm is investigated. Thereby the simulations were considered until localization and softening on the material scale occurred. It is shown that the proposed method presents an effective way to determine the exact physical macroscopic response considering arbitrary microstructures and loading conditions.

**MSC:**

[74A40](#) Random materials and composite materials  
[65T50](#) Numerical methods for discrete and fast Fourier transforms  
[74Q05](#) Homogenization in equilibrium problems of solid mechanics  
[74R05](#) Brittle damage

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[multiscale analysis](#); [computational homogenization](#); [continuum damage mechanics](#); [fast Fourier transforms](#); [composite materials](#)

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