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An enhanced beam-theory model of the mixed-mode bending (MMB) test. I: Literature review and mechanical model. (English) [Zbl 1293.74256](#)

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Summary: The paper presents a mechanical model of the mixed-mode bending (MMB) test used to assess the mixed-mode interlaminar fracture toughness of composite laminates. The laminated specimen is considered as an assemblage of two sublaminates partly connected by an elastic-brittle interface. The problem is formulated through a set of 36 differential equations, accompanied by suitable boundary conditions. Solution of the problem is achieved by separately considering the two subproblems related to the symmetric and antisymmetric parts of the loads, which for symmetric specimens correspond to fracture modes I and II, respectively. Explicit expressions are determined for the interfacial stresses, internal forces, and displacements.

MSC:

74K10 Rods (beams, columns, shafts, arches, rings, etc.)

74-05 Experimental work for problems pertaining to mechanics of deformable solids

74E30 Composite and mixture properties

74-02 Research exposition (monographs, survey articles) pertaining to mechanics of deformable solids

Cited in **1** Review
Cited in **3** Documents

Keywords:

mixed-mode bending (MMB) test; beam-theory model; analytical solution; laminated composite; interlaminar fracture toughness; delamination

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References:

- [1] Garg AC (1988) Delamination—a damage mode in composite structures. *Eng Fract Mech* 29(5):557–584. doi: 10.1016/0013-7944(88)90181-6 · doi:10.1016/0013-7944(88)90181-6
- [2] Sela N, Ishai O (1989) Interlaminar fracture toughness and toughening of laminated composite materials: a review. *Composites* 20(5):423–435. doi: 10.1016/0010-4361(89)90211-5 · doi:10.1016/0010-4361(89)90211-5
- [3] Tay TE (2003) Characterization and analysis of delamination fracture in composites: an overview of developments from 1990 to 2001. *Appl Mech Rev* 56(1):1–31. doi: 10.1115/1.1504848 · doi:10.1115/1.1504848
- [4] Friedrich K (ed) (1989) *Application of fracture mechanics to composite materials*. Elsevier, Amsterdam
- [5] Adams DF, Carlsson LA, Pipes RB (2003) *Experimental characterization of advanced composite materials*, 3rd edn. CRC Press, Boca Raton · Zbl 1067.74003
- [6] Brunner AJ, Blackman BRK, Davies P (2008) A status report on delamination resistance testing of polymer–matrix composites. *Eng Fract Mech* 75(9):2779–2794. doi: 10.1016/j.engfracmech.2007.03.012 · doi:10.1016/j.engfracmech.2007.03.012
- [7] Crews JH Jr, Reeder JR (1988) A mixed-mode bending apparatus for delamination testing. NASA TM-100662. <http://ntrs.nasa.gov/archive/nasa/>
- [8] Reeder JR, Crews JH Jr (1990) Mixed-mode bending method for delamination testing. *AIAA J* 28(7):1270–1276. doi: 10.2514/3.25204 · doi:10.2514/3.25204
- [9] Reeder JR, Crews JH Jr (1991) Nonlinear analysis and redesign of the mixed-mode bending delamination test. NASA TM-102777. http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19910010169_1991010169.pdf
- [10] Reeder JR, Crews JH Jr (1992) Redesign of the mixed-mode bending delamination test to reduce nonlinear effects. *J Compos Technol Res* 14(1):12–19. doi: 10.1520/CTR10078J · doi:10.1520/CTR10078J
- [11] Reeder JR (1992) An evaluation of mixed-mode delamination failure criteria. NASA TM-104210. <http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/>
- [12] Reeder JR (2003) Refinements to the mixed-mode bending test for delamination toughness. *J Compos Technol Res* 25(4):191–195. doi: 10.1520/CTR10961J
- [13] ASTM (2006) Standard test method for mixed mode I–mode II interlaminar fracture toughness of unidirectional fiber reinforced polymer matrix composites. D6671/D6671M-06. American Society for Testing and Materials, West Conshohocken. doi: 10.1520/D6671_D6671M-06

- [14] Allix O, Ladevèze P (1992) Interlaminar interface modelling for the prediction of delamination. *Compos Struct* 22(4):235–242. doi: 10.1016/0263-8223(92)90060-P · doi:10.1016/0263-8223(92)90060-P
- [15] Corigliano A (1993) Formulation, identification and use of interface models in the numerical analysis of composite delamination. *Int J Solids Struct* 30(20):2779–2811. doi: 10.1016/0020-7683(93)90154-Y · Zbl 0782.73055 · doi:10.1016/0020-7683(93)90154-Y
- [16] Bennati S, Colleluori M, Corigliano D, Valvo PS (2009) An enhanced beam-theory model of the asymmetric double cantilever beam (ADCB) test for composite laminates. *Compos Sci Technol* 69(11–12):1735–1745. doi: 10.1016/j.compscitech.2009.01.019 · doi:10.1016/j.compscitech.2009.01.019
- [17] Bennati S, Fiscaro P, Valvo PS (2013) An enhanced beam-theory model of the mixed-mode bending (MMB) test—Part II: Applications and results. *Meccanica*. doi: 10.1007/s11012-012-9682-7 · Zbl 1293.74258
- [18] Chen JH, Sernow R, Schultz E, Hinrichsen G (1999) A modification of the mixed-mode bending test apparatus. *Composites, Part A, Appl Sci Manuf* 30(7):871–877. doi: 10.1016/S1359-835X(98)00193-6 · doi:10.1016/S1359-835X(98)00193-6
- [19] Kinloch AJ, Wang Y, Williams JG, Yayla P (1993) The mixed-mode delamination of fibre composite materials. *Compos Sci Technol* 47(3):225–237. doi: 10.1016/0266-3538(93)90031-B · doi:10.1016/0266-3538(93)90031-B
- [20] Kenane M, Benzeggagh ML (1997) Mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites under fatigue loading. *Compos Sci Technol* 57(5):597–605. doi: 10.1016/S0266-3538(97)00021-3 · doi:10.1016/S0266-3538(97)00021-3
- [21] Yum Y-J, You H (2001) Pure mode I, II and mixed mode interlaminar fracture of graphite/epoxy composite materials. *J Reinf Plast Compos* 20(9):794–808. doi: 10.1177/073168401772678571 · doi:10.1177/073168401772678571
- [22] Soboyejo WO, Lu G-Y, Chengalva S, Zhang J, Kenner V (1999) A modified mixed-mode bending specimen for the interfacial fracture testing of dissimilar materials. *Fatigue Fract Eng Mater Struct* 22(9):799–810. doi: 10.1046/j.1460-2695.1999.00203.x · doi:10.1046/j.1460-2695.1999.t01-1-00203.x
- [23] Marannano GV, Pasta A (2007) An analysis of interface delamination mechanisms in orthotropic and hybrid fiber-metal composite laminates. *Eng Fract Mech* 74(4):612–626. doi: 10.1016/j.engfracmech.2006.09.004 · doi:10.1016/j.engfracmech.2006.09.004
- [24] Suárez JC, López F, Miguel S, Pinilla P, Herreros MA (2009) Determination of the mixed-mode fracture energy of elastomeric structural adhesives: evaluation of debonding buckling in fibre-metal hybrid laminates. *Fatigue Fract Eng Mater Struct* 32(2):127–140. doi: 10.1111/j.1460-2695.2008.01317.x · doi:10.1111/j.1460-2695.2008.01317.x
- [25] Bhashyan S, Davidson BD (1997) Evaluation of data reduction methods for the mixed mode bending test. *AIAA J* 35(3):546–552. doi: 10.2514/2.129 · doi:10.2514/2.129
- [26] Kanninen MF (1973) An augmented double cantilever beam model for studying crack propagation and arrest. *Int J Fract* 9(1):83–92. doi: 10.1007/BF00035958
- [27] Carlsson LA, Gillespie JW, Pipes RB (1986) On the analysis and design of the end notched flexure (ENF) specimen for mode II testing. *J Compos Mater* 20(6):594–604. doi: 10.1177/002199838602000606 · doi:10.1177/002199838602000606
- [28] Fan C, Ben Jar P-Y, Cheng J-JR (2006) Revisit the analysis of end-notched-flexure (ENF) specimen. *Compos Sci Technol* 66(10):1497–1498. doi: 10.1016/j.compscitech.2006.01.016 · doi:10.1016/j.compscitech.2006.01.016
- [29] Valvo PS (2008) Does shear deformability influence the mode II delamination of laminated beams? In: ECF 17–17th European Conference on Fracture, 2–5 September 2008, Brno, Czech Republic
- [30] Williams JG (1989) End corrections for orthotropic DCB specimens. *Compos Sci Technol* 35(4):367–376. doi: 10.1016/0266-3538(89)90058-4 · doi:10.1016/0266-3538(89)90058-4
- [31] Hashemi S, Kinloch AJ, Williams JG (1990) The analysis of interlaminar fracture in uniaxial fibre-polymer composites. *Proc R Soc Lond. Ser A* 427(1872):173–199. doi: 10.1098/rspa.1990.0007 · doi:10.1098/rspa.1990.0007
- [32] Wang Y, Williams JG (1992) Corrections for mode II fracture toughness specimens of composites materials. *Compos Sci Technol* 43(3):251–256. doi: 10.1016/0266-3538(92)90096-L · doi:10.1016/0266-3538(92)90096-L
- [33] Wang JL, Qiao PZ (2004) Novel beam analysis of end notched flexure specimen for mode-II fracture. *Eng Fract Mech* 71(2):219–231. doi: 10.1016/S0013-7944(03)00096-1 · doi:10.1016/S0013-7944(03)00096-1
- [34] de Morais AB (2011) Novel cohesive beam model for the end-notched flexure (ENF) specimen. *Eng Fract Mech* 78(17):3017–3029. doi: 10.1016/j.engfracmech.2011.08.019 · doi:10.1016/j.engfracmech.2011.08.019
- [35] Jumel J, Budzik MK, Ben Salem N, Shanahan MER (2013) Instrumented end notched flexure—crack propagation and process zone monitoring. Part I: Modelling and analysis. *Int J Solids Struct* 50(2):310–319. doi: 10.1016/j.ijsolstr.2012.08.028 · doi:10.1016/j.ijsolstr.2012.08.028
- [36] de Morais AB, Pereira AB (2006) Mixed mode I + II interlaminar fracture of glass/epoxy multidirectional laminates—Part 1: Analysis. *Compos Sci Technol* 66(13):1889–1895. doi: 10.1016/j.compscitech.2006.04.006 · doi:10.1016/j.compscitech.2006.04.006
- [37] Pereira AB, de Morais AB (2006) Mixed mode I + II interlaminar fracture of glass/epoxy multidirectional laminates—Part 2: Experiments. *Compos Sci Technol* 66(13):1896–1902. doi: 10.1016/j.compscitech.2006.04.008 · doi:10.1016/j.compscitech.2006.04.008
- [38] de Morais AB, Pereira AB (2007) Interlaminar fracture of multidirectional glass/epoxy laminates under mixed-mode I + II loading. *Mech Compos Mater* 43(3):233–244. doi: 10.1007/s11029-007-0023-1 · doi:10.1007/s11029-007-0023-1
- [39] Pereira AB, de Morais AB (2008) Mixed mode I + II interlaminar fracture of carbon/epoxy laminates. *Composites, Part A* 39(2):322–333. doi: 10.1016/j.compositesa.2007.10.013 · doi:10.1016/j.compositesa.2007.10.013
- [40] Ducept F, Davies P, Gamby D (1997) An experimental study to validate tests used to determine mixed mode failure criteria of glass/epoxy composites. *Composites, Part A* 28(8):719–729. doi: 10.1016/S1359-835X(97)00012-2 · doi:10.1016/S1359-835X(97)00012-2

- [41] Ducept F, Gamby D, Davies P (1999) A mixed-mode failure criterion derived from tests on symmetric and asymmetric specimens. *Compos Sci Technol* 59(4):609–619. doi: 10.1016/S0266-3538(98)00105-5 · doi:10.1016/S0266-3538(98)00105-5
- [42] Ducept F, Davies P, Gamby D (2000) Mixed mode failure criteria for a glass/epoxy composite and an adhesively bonded composite/composite joint. *Int J Adhes Adhes* 20(3):233–244. doi: 10.1016/S0143-7496(99)00048-2 · doi:10.1016/S0143-7496(99)00048-2
- [43] Benzeggagh ML, Kenane M (1996) Measurement of mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites with mixed-mode bending apparatus. *Compos Sci Technol* 56(4):439–449. doi: 10.1016/0266-3538(96)00005-X · doi:10.1016/0266-3538(96)00005-X
- [44] Martin RH, Hansen PL (1997) Experimental compliance calibration for the MMB specimen. In: Armanios EA (ed) *Composite materials: fatigue and fracture*, vol 6. ASTM STP, vol 1285, pp 305–323. doi: 10.1520/STP19934S
- [45] Ozdil F, Carlsson LA (1999) Beam analysis of angle-ply laminate mixed-mode bending specimens. *Compos Sci Technol* 59(6):937–945. doi: 10.1016/S0266-3538(98)00128-6 · doi:10.1016/S0266-3538(98)00128-6
- [46] Kim BW, Mayer AH (2003) Influence of fiber direction and mixed-mode ratio on delamination fracture toughness of carbon/epoxy laminates. *Compos Sci Technol* 63(5):695–713. doi: 10.1016/S0266-3538(02)00258-0 · doi:10.1016/S0266-3538(02)00258-0
- [47] Yokozeki T, Ogasawara T, Aoki T (2008) Correction method for evaluation of interfacial fracture toughness of DCB, ENF and MMB specimens with residual thermal stresses. *Compos Sci Technol* 68(3–4):760–767. doi: 10.1016/j.compscitech.2007.08.025 · doi:10.1016/j.compscitech.2007.08.025
- [48] Jagan U, Chauhan PS, Parameswaran V (2008) Energy release rate for interlaminar cracks in graded laminates. *Compos Sci Technol* 68(6):1480–1488. doi: 10.1016/j.compscitech.2007.10.027 · doi:10.1016/j.compscitech.2007.10.027
- [49] Quispitupa A, Berggreen C, Carlsson LA (2009) On the analysis of a mixed mode bending sandwich specimen for debond fracture characterization. *Eng Fract Mech* 76(4):594–613. doi: 10.1016/j.engfracmech.2008.12.008 · doi:10.1016/j.engfracmech.2008.12.008
- [50] Allix O, Corigliano A (1996) Modeling and simulation of crack propagation in mixed-modes interlaminar fracture specimens. *Int J Fract* 77(2):111–140. doi: 10.1007/BF00037233 · doi:10.1007/BF00037233
- [51] Miravete A, Jiménez MA (2002) Application of the finite element method to prediction of onset of delamination growth. *Appl Mech Rev* 55(2):89–105. doi: 10.1115/1.1450763 · doi:10.1115/1.1450763
- [52] Jiménez MA, Miravete A (2004) Application of the finite-element method to predict the onset of delamination growth. *J Compos Mater* 38(15):1309–1335. doi: 10.1177/0021998304042734 · doi:10.1177/0021998304042734
- [53] Camanho PP, Dávila CG, de Moura NF (2003) Numerical simulation of mixed-mode progressive delamination in composite materials. *J Compos Mater* 37(16):1415–1438. doi: 10.1177/0021998303034505 · doi:10.1177/0021998303034505
- [54] Turon A, Camanho PP, Costa J, Dávila CG (2006) A damage model for the simulation of delamination in advanced composites under variable-mode loading. *Mech Mater* 38(11):1072–1089. doi: 10.1016/j.mechmat.2005.10.003 · doi:10.1016/j.mechmat.2005.10.003
- [55] Tumino D, Cappello F (2007) Simulation of fatigue delamination growth in composites with different mode mixtures. *J Compos Mater* 41(20):2415–2441. doi: 10.1177/0021998307075439 · doi:10.1177/0021998307075439
- [56] Oliveira JM, de Moura MFSF, Silva MAL, Morais JLL (2007) Numerical analysis of the MMB test for mixed-mode I/II wood fracture. *Compos Sci Technol* 67(2):1764–1771. doi: 10.1016/j.compscitech.2006.11.007 · doi:10.1016/j.compscitech.2006.11.007
- [57] de Moura MFSF, Oliveira JM, Morais JLL, Xavier J (2010) Mixed-mode I/II wood fracture characterization using the mixed-mode bending test. *Eng Fract Mech* 77(1):144–152. doi: 10.1016/j.engfracmech.2009.09.014 · doi:10.1016/j.engfracmech.2009.09.014
- [58] Warrior NA, Pickett AK, Lourenço NSF (2003) Mixed-mode delamination—experimental and numerical studies. *Strain* 39(4):153–159. doi: 10.1046/j.1475-1305.2003.00088.x · doi:10.1046/j.1475-1305.2003.00088.x
- [59] Iannucci L (2006) Dynamic delamination modelling using interface elements. *Comput Struct* 84(15–16):1029–1048. doi: 10.1016/j.compstruc.2006.02.002 · doi:10.1016/j.compstruc.2006.02.002
- [60] Borg R, Nilsson L, Simonsson K (2004) Simulating DCB, ENF and MMB experiments using shell elements and a cohesive zone model. *Compos Sci Technol* 64(2):269–278. doi: 10.1016/S0266-3538(03)00255-0 · doi:10.1016/S0266-3538(03)00255-0
- [61] Aymerich F, Lecca G, Priolo P (2007) Modelling of delamination growth in composite laminates by the virtual internal bond method. *Composites, Part A* 39(2):145–153. doi: 10.1016/j.compositesa.2007.11.012 · doi:10.1016/j.compositesa.2007.11.012
- [62] van der Meer FP, Sluys LJ (2009) A phantom node formulation with mixed mode cohesive law for splitting in laminates. *Int J Fract* 158(2):107–124. doi: 10.1007/s10704-009-9344-5 · Zbl 1400.74106 · doi:10.1007/s10704-009-9344-5
- [63] Blanco N, Turon A, Costa J (2006) An exact solution for the determination of the mode mixture in the mixed-mode bending delamination test. *Compos Sci Technol* 66(10):1256–1258. doi: 10.1016/j.compscitech.2005.10.028 · doi:10.1016/j.compscitech.2005.10.028
- [64] Tenchev RT, Falzon BG (2007) A correction to the analytical solution of the mixed-mode bending (MMB) problem. *Compos Sci Technol* 67(3–4):662–668. doi: 10.1016/j.compscitech.2006.05.007 · doi:10.1016/j.compscitech.2006.05.007
- [65] Massabò R, Cox BN (2001) Unusual characteristics of mixed-mode delamination fracture in the presence of large-scale bridging. *Mech Compos Mater Struct* 8(1):61–80. doi: 10.1080/107594101459833 · doi:10.1080/107594101459833
- [66] Szekrényes A, Uj J (2006) Comparison of some improved solutions for mixed-mode composite delamination coupons. *Comput Struct* 72(3):321–329. doi: 10.1016/j.compstruct.2005.01.002 · doi:10.1016/j.compstruct.2005.01.002
- [67] Szekrényes A (2007) Improved analysis of unidirectional composite delamination specimens. *Mech Mater* 39(10):953–974. doi: 10.1016/j.mechmat.2007.04.002 · Zbl 1121.74468 · doi:10.1016/j.mechmat.2007.04.002
- [68] Jones RM (1999) *Mechanics of composite materials*, 2nd edn. Taylor & Francis, Philadelphia
- [69] Vannucci P, Verchery G (2001) A special class of uncoupled and quasi-homogeneous laminates. *Compos Sci Technol* 61(10):1465–1473. doi: 10.1016/S0266-3538(01)00039-2 · doi:10.1016/S0266-3538(01)00039-2

- [70] Timoshenko SP (1984) Strength of materials: elementary theory and problems, vol 1. Krieger, Melbourne
- [71] Cotterell B, Hbaieb K, Williams JG, Hadavinia H, Tropsa V (2006) The root rotation in double cantilever beam and peel tests. *Mech Mater* 38(7):571–584. doi: 10.1016/j.mechmat.2005.11.001 · doi:10.1016/j.mechmat.2005.11.001
- [72] Andrews MG, Massabò R (2007) The effects of shear and near tip deformations on energy release rate and mode mixity of edge-cracked orthotropic layers. *Eng Fract Mech* 74(17):2700–2720. doi: 10.1016/j.engfracmech.2007.01.013 · doi:10.1016/j.engfracmech.2007.01.013

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