

Cuccagna, Scipio

On asymptotic stability of moving ground states of the nonlinear Schrödinger equation.

(English) [Zbl 1293.35289](#)

Trans. Am. Math. Soc. 366, No. 6, 2827-2888 (2014).

Summary: We extend to the case of moving solitons, the result on asymptotic stability of ground states of the NLS obtained by the author in an earlier paper. For technical reasons we consider only smooth solutions. The proof is similar to the earlier paper. However now the flows required for the Darboux Theorem and the Birkhoff normal forms, instead of falling within the framework of standard theory of ODE's, are related to quasilinear hyperbolic symmetric systems. It is also not obvious that the Darboux Theorem can be applied, since we need to compare two symplectic forms in a neighborhood of the ground states not in $H^1(\mathbb{R}^3)$, but rather in the space Σ where also the variance is bounded. But the NLS does not preserve small neighborhoods of the ground states in Σ .

MSC:

[35Q55](#) NLS equations (nonlinear Schrödinger equations)

[35B40](#) Asymptotic behavior of solutions to PDEs

[35Q51](#) Soliton equations

[37K40](#) Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems

Cited in **17** Documents

Keywords:

nonlinear Schrödinger equation; solitons; asymptotic stability; Darboux theorem; Birkhoff normal forms

Full Text: [DOI](#) [arXiv](#)

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