Kaptan, D. A.

A generalization of the Goldston-Pintz-Yildirim prime gaps result to number fields. (English)

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Let p_n be the nth prime. A celebrated result of Goldston-Pintz-Yildirim is that

$$\liminf_{n\to\infty} \frac{p_{n+1}-p_n}{\log p_n} = 0.$$

In the paper under review, the author extends this result to the set-up of totally real number fields. The result is that if $\mathbb{K}$ is a totally real number field, then

$$\liminf_{\substack{\omega_0,\omega_1\in\mathcal{O}_{\mathbb{K}} \text{ primes}\\ \omega_0\neq\omega_1}} \frac{N_{\mathbb{K}/\mathbb{Q}}(\omega_1-\omega_0)}{N_{\mathbb{K}/\mathbb{Q}}(\omega_0)} = 0.$$

A key ingredient in the proof of the Goldston-Pintz-Yildirim result is the Bombieri-Vinogradiv theorem. The present work follows closely that proof with the Bombieri-Vinogradov theorem replaced by a generalization of it due to Hintz to the case of totally real number fields.

Reviewer: Florian Luca (Morelia)

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11R80 Totally real fields
11R45 Density theorems

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