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A generalization of the Goldston-Pintz-Yildirim prime gaps result to number fields. (English)

Zbl 1291.11133

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Let p_n be the n th prime. A celebrated result of Goldston-Pintz-Yildirim is that

$$\liminf_{n \rightarrow \infty} \frac{p_{n+1} - p_n}{\log p_n} = 0.$$

In the paper under review, the author extends this result to the set-up of totally real number fields. The result is that if \mathbb{K} is a totally real number field, then

$$\liminf_{\substack{\omega_0, \omega_1 \in \mathcal{O}_{\mathbb{K}} \text{ primes} \\ \omega_0 \neq \omega_1}} \frac{N_{\mathbb{K}/\mathbb{Q}}(\omega_1 - \omega_0)}{N_{\mathbb{K}/\mathbb{Q}}(\omega_0)} = 0.$$

A key ingredient in the proof of the Goldston-Pintz-Yildirim result is the Bombieri-Vinogradov theorem. The present work follows closely that proof with the Bombieri-Vinogradov theorem replaced by a generalization of it due to Hintz to the case of totally real number fields.

Reviewer: [Florian Luca \(Morelia\)](#)

MSC:

11R80 Totally real fields

11R45 Density theorems

Cited in **1** Review
Cited in **2** Documents

Keywords:

distribution of primes; algebraic number field

Full Text: [DOI](#)

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