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Single-crossover recombination and ancestral recombination trees. (English) Zbl 1284.92063

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Summary: We consider the Wright-Fisher model for a population of N individuals, each identified with a sequence of a finite number of sites, and single-crossover recombinations between them. We trace back the ancestry of single individuals from the present population. In the $N \to \infty$ limit without rescaling of parameters or time, this ancestral process is described by a random tree, whose branching events correspond to the splitting of the sequence due to recombination. With the help of a decomposition of the trees into subtrees, we calculate the probabilities of the topologies of the ancestral trees. At the same time, these probabilities lead to a semi-explicit solution of the deterministic single-crossover equation. The latter is a discrete-time dynamical system that emerges from the Wright-Fisher model via a law of large numbers and has been waiting for a solution for many decades.

MSC:

92D15 Problems related to evolution
92D10 Genetics and epigenetics
60J28 Applications of continuous-time Markov processes on discrete state spaces
60F15 Strong limit theorems

Keywords:

population genetics; segmentation processes; ancestral trees; subtree decompositions

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