Cardaliaguet, Pierre; Souganidis, Panagiotis E.
Homogenization and enhancement of the G-equation in random environments. (English)
Zbl 1284.60126
Commun. Pure Appl. Math. 66, No. 10, 1582-1628 (2013).

The homogenization limit when the perturbation parameter tends to zero can be thought of as a small parameter perturbation technique. Here one applies this method to a G-equation submitted to a general stationary ergodic environment. The framework of the study is the so-called viscosity solution. The averaging properties of this G-equation cannot be studied by using subadditive theorem which is the standard approach to the homogenization of Hamilton-Jacobi equation in random media. The main contribution of the paper is to propose a new approach to circumvent these problems, which reduces to a controllability estimate and the construction of a random sequence which defines a long-time asymptotic limit.

Reviewer: Guy Jumarie (Montréal)

MSC:

60H15 Stochastic partial differential equations (aspects of stochastic analysis)
35R60 PDEs with randomness, stochastic partial differential equations
60K37 Processes in random environments

Keywords:

G-equation; controllability; homogenization; perturbation; viscosity; small parameter

Full Text: DOI

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