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Local controllability of 1D Schrödinger equations with bilinear control and minimal time. (English) Zbl 1281.93016

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Summary: We consider a linear Schrödinger equation, on a bounded interval, with bilinear control.

In *K.Beauchard* and *C. Laurent* ["Local controllability of 1D linear and nonlinear Schrödinger equations with bilinear control", J. Math. Pures Appl., 94, 520-554 (2010; Zbl 1202.35332)], the authors prove that, under an appropriate non-degeneracy assumption, this system is controllable, locally around the ground state, in arbitrary time. In *J.-M. Coron* ["On the small-time local controllability of a quantum particle in a moving one-dimensional infinite square potential well". C. R. Acad. Sciences Paris, Ser. I, 342 , 103-108 (2006; Zbl 1082.93002)], the author proves that a positive minimal time is required for this controllability result, on a particular degenerate example.

In this article, we propose a general context for the local controllability to hold in large time, but not in small time. The existence of a positive minimal time is closely related to the behavior of the second order term, in the power series expansion of the solution.

MSC:

93B05 Controllability
93C20 Control/observation systems governed by partial differential equations
81Q93 Quantum control

Keywords:

exact controllability; Schrödinger equation; bilinear control; minimal time; power series expansion

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