Kittaneh, Fuad; **Krnić, Mario**; **Lovričević, Neda**; **Pečarić, Josip**; **Hirzallah, Omar**
Refinements and reverses of means inequalities for Hilbert space operators. (English)
Zbl 1276.47021
Banach J. Math. Anal. 7, No. 2, 15-29 (2013).

This paper discusses improvements of a series of inequalities for Hilbert space operators.

The authors define a functional that measures the difference between the classical arithmetic and geometric means and also deduce some significant scalar inequalities. Section 3 provides, under certain conditions, improvements of the series of inequalities and establishes the lower bound for the difference between the arithmetic and geometric operator means. As an application, in Section 4, the authors establish an improved variant of an inequality concerning the Heinz operator mean. In the final section, the authors present some eigenvalue inequalities for differences of certain operator means.

Reviewer: V. Lokesha (Bangalore)

MSC:

47A63 Linear operator inequalities
47A10 Spectrum, resolvent
47B06 Riesz operators; eigenvalue distributions; approximation numbers, *s*-numbers, Kolmogorov numbers, entropy numbers, etc. of operators
47B07 Linear operators defined by compactness properties
47B15 Hermitian and normal operators (spectral measures, functional calculus, etc.)
26D20 Other analytical inequalities

Keywords:

positive operator; compact operator; operator mean; refinement; eigenvalue

References:

[1] R. Bhatia, Matrix Analysis , GTM169, Springer-Verlag, New York, 1997. · Zbl 0863.15001

[2] V. Cirtoaje, The best lower bound depended on two fixed variables for Jensen's inequality with ordered variables , J. Inequal. Appl. 2010 , Art. ID 128258, 12 pp. · Zbl 1204.26031 · doi:10.1155/2010/128258

[3] V. Cirtoaje, The best lower bound for Jensen's inequality with three fixed ordered variables , Banach J. Math. Anal. 7 (2013), no. 1. · Zbl 1266.26026

[4] T. Furuta, J. Mićić Hot, J. Pečarić and Y. Seo, Mond-Pečarić Method in Operator Inequalities , Element, Zagreb, 2005.

[5] I.C. Gohberg and M.G. Krein, Introduction to the Theory of Linear Nonself-adjoint Operators , Transl. Math. Monographs, vol. 18, AMS, Providence, RI, 1969. · Zbl 0181.13504

[6] O. Hirzallah and F. Kittaneh, Norm inequalities for weighted power means of operators , Linear Algebra Appl. 341 (2002), Special issue dedicated to Professor T. Ando, 181-193. · Zbl 1017.47003 · doi:10.1016/S0024-3795(01)00377-9

[7] O. Hirzallah, F. Kittaneh, M. Krnić, N. Lovričević and J. Pečarić, Eigenvalue inequalities for differences of means of Hilbert space operators , Linear Algebra Appl. 436 (2012), no. 5, 1516-1527. · Zbl 1248.47019 · doi:10.1016/j.laa.2011.08.037

[8] F. Kittaneh and Y. Manasrah, Improved Young and Heinz inequalities for matrices , J. Math. Anal. Appl. 361 (2010), no. 1, 262-269. · Zbl 1180.15021 · doi:10.1016/j.jmaa.2009.08.059

[9] F. Kittaneh and Y. Manasrah, Reverse Young and Heinz inequalities for matrices , Linear Multilinear Algebra 59 (2011), no. 9, 1031-1037. · Zbl 1225.15022 · doi:10.1080/03081087.2010.551661

[10] F. Kittaneh, M. Krnić, N. Lovričević and J. Peč arić, Improved arithmetic-geometric and Heinz means inequalities for Hilbert space operators , Publ. Math. Debrecen 80 (2012), no. 3-4, 465-478. · Zbl 1275.47038 · doi:10.5486/PMD.2012.5193

[11] J. Mićić, J. Pečarić and V. Šimić, Inequalities involving the arithmetic and geometric means , Math. Inequal. Appl. 11 (2008), no. 3, 415-430. · Zbl 1154.47013

[12] J.R. Ringrose, Compact Non-self-adjoint Operators , Van Nostrand Reinhold, New York, 1971. · Zbl 0223.47012