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Solutions to nonlinear Schrödinger equations with singular electromagnetic potential and critical exponent. (English) Zbl 1273.35247

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The authors consider singular (stationary) Schrödinger equations with a magnetic potential and a critical nonlinearity of the following form:

$$\left(\mathrm{i}\nabla - \frac{A(\theta)}{|x|}\right)^2 u - \frac{a}{|x|^2} u = |u|^{2^*-2} u \qquad \text{in } \mathbb{R}^N \setminus \{0\}. \tag{1}$$

Here it is assumed that $N \geq 4$, and $2^* := 2N/(N-2)$ denotes the usual critical Sobolev exponent. Moreover, $A \in L^\infty(\mathbb{S}^{N-1}, \mathbb{R}^N)$ is assumed to be equivariant under the action of $G := \mathrm{SO}(2) \times \mathrm{SO}(N-2)$. The main result states that there is $a^* < 0$ such that (1) possesses two solutions in $D^{1,2}(\mathbb{R}^N)$ for every $a < a^*$, one invariant under the action of G (i.e., biradially symmetric), and one invariant under $\mathbb{Z}_k \times \mathrm{SO}(N-2)$ for some $k \in \mathbb{N}$.

An analogous result holds for magnetic Aharonov-Bohm type potentials of the form

$$\mathcal{A}(x_1, x_2, x_3) := \left(\frac{-\alpha x_2}{x_1^2 + x_2^2}, \frac{\alpha x_1}{x_1^2 + x_2^2}, 0\right),$$

where $(x_1, x_2) \in \mathbb{R}^2$ and $x_3 \in \mathbb{R}^{N-2}$.

Reviewer: Nils Ackermann (Mexico City)

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)

35J75 Singular elliptic equations

35J91 Semilinear elliptic equations with Laplacian, bi-Laplacian or poly-Laplacian

35J20 Variational methods for second-order elliptic equations

35B06 Symmetries, invariants, etc. in context of PDEs

Keywords:

singular electromagnetic potentials; Hardy's inequality; nonlinear Schrödinger equations

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References:

[1] L. Abatangelo and S. Terracini, A note on the complete rotational invariance of biradial solutions to semilinear elliptic equations. Preprint, 2009. · Zbl 1218.35105

[2] Arioli G., Szulkin A.: A semilinear Schrödinger equation in the presence of a magnetic field. Arch. Ration. Mech. Anal. 170, 277–295 (2003) · Zbl 1051.35082 · doi:10.1007/s00205-003-0274-5

[3] Bartsch T., Dancer E.N., Peng S.: On multi-bump semi-classical bound states of nonlinear Schrödinger equations with electromagnetic fields. Adv. Differential Equations 11, 781–812 (2006) · Zbl 1146.35081

[4] Bahri A., Li Y.Y.: On a min-max procedure for the existence of positive solutions for certain scalar field equations in \$\$\{\(\backslash\)mathbb\{R\}\^N\}\$\$. Rev. Mat. Iberoamericana 6, 1–15 (1990) · Zbl 0731.35036 · doi:10.4171/RMI/92

[5] Bourgain J., Brezis H.: New estimates for elliptic equations and Hodge type systems. J. Eur. Math. Soc. (JEMS) 9(2), 277–315 (2007) · Zbl 1176.35061 · doi:10.4171/JEMS/80

[6] Brezis H., Nirenberg L.: Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents. Comm. Pure Appl. Math. 36, 437–477 (1983) · Zbl 0541.35029 · doi:10.1002/cpa.3160360405

[7] Chabrowski J., Szulkin A.: On the Schrödinger equation involving a critical Sobolev exponent and magnetic field. Topol. Methods Nonlinear Anal. 25, 3–21 (2005) · Zbl 1176.35022

[8] Cingolani S.: Semiclassical stationary states of nonlinear Schrödinger equations with an external magnetic field. J. Differential Equations 188, 52–79 (2003) · Zbl 1062.81056 · doi:10.1016/S0022-0396(02)00058-X

[9] Cingolani S., Clapp M.: Intertwining semiclassical bound states to a nonlinear magnetic Schrödinger equation. Nonlinearity 22, 2309–2331 (2009) · Zbl 1173.35678 · doi:10.1088/0951-7715/22/9/013

[10] Cingolani S., Secchi S.: Semiclassical limit for nonlinear Schrödinger equations with electromagnetic fields. J. Math. Anal. Appl. 275, 108–130 (2002) · Zbl 1014.35087 · doi:10.1016/S0022-247X(02)00278-0

[11] Cingolani S., Secchi S.: Semiclassical states for NLS equations with magnetic potentials having polynomial growths. J. Math. Phys. 46, 053503 (2005) · Zbl 1110.81081 · doi:10.1063/1.1874333

[12] Clapp M., Iturriaga R., Szulkin A.: Periodic and Bloch solutions to a magnetic nonlinear Schrödinger equation. Adv. Nonlinear Stud. 9, 639–655 (2009) · Zbl 1182.35203

[13] Clapp M., Szulkin A.: Multiple solutions to a nonlinear Schrödinger equation with Aharonov-Bohm magnetic potential. Nonlinear Differential Equations Appl. 17, 229–248 (2010) · Zbl 1189.35302 · doi:10.1007/s00030-009-0051-8

[14] L. C. Evans, Partial differential equations. 2nd ed., Grad. Stud. Math. 19, American Mathematical Society, Providence, RI, 2010. · Zbl 1194.35001

[15] M. Esteban and P. L. Lions, Stationary solutions of nonlinear Schrödinger equations with an external magnetic field. In: Partial Differential Equations and the Calculus of Variations, Vol. I, Progr. Nonlinear Differential Equations Appl. 1, Birkhäuser Boston, Boston, MA, 1989, 401–449.

[16] Felli V., Ferrero A., Terracini S.: Asymptotic behavior of solutions to Schrödinger equations near an isolated singularity of the electromagnetic potential. J. Eur. Math. Soc. (JEMS) 13, 119–174 (2011) · Zbl 1208.35070 · doi:10.4171/JEMS/246

[17] Felli V., Marchini E.M., Terracini S.: On the behavior of solutions to Schrödinger equations with dipole-type potentials near the singularity. Discrete Contin. Dyn. Syst. 21, 91–119 (2008) · Zbl 1141.35362 · doi:10.3934/dcds.2008.21.91

[18] Kurata K.: A unique continuation theorem for the Schrödinger equation with singular magnetic field. Proc. Amer. Math. Soc. 125, 853–860 (1997) · Zbl 0887.35026 · doi:10.1090/S0002-9939-97-03672-1

[19] Kurata K.: Existence and semi-classical limit of the least energy solution to a nonlinear Schrödinger equation with electromagnetic fields. Nonlinear Anal. 41, 763–778 (2000) · Zbl 0993.35081 · doi:10.1016/S0362-546X(98)00308-3

[20] A. Laptev and T. Weidl, Hardy inequalities for magnetic Dirichlet forms. In: Mathematical Results in Quantum Mechanics (Prague, 1998), Oper. Theory Adv. Appl. 108, Birkhäuser, Basel, 1999, 299–305. · Zbl 0977.26005

[21] Leinfelder H.: Gauge invariance of Schrödinger operators and related spectral properties. J. Operator Theory 9, 163–179 (1983) · Zbl 0528.35024

[22] Pankov A.A.: On nontrivial solutions of the nonlinear Schrödinger equation with a magnetic field. Funct. Anal. Appl. 37, 75–77 (2003) · Zbl 1028.35142 · doi:10.1023/A:1022984313164

[23] G. Rozenblum and M. Melgaard, Schrödinger operators with singular potentials. In: Stationary Partial Differential Equations, Vol. II, Handb. Differ. Equ., Elsevier/North–Holland, Amsterdam, 2005, 407–517. · Zbl 1206.35084

[24] Reed M., Simon B.: Methods of modern mathematical physics. II. Fourier analysis, self-adjointness. Academic Press, New York (1975) · Zbl 0308.47002

[25] Schindler I., Tintarev K.: A nonlinear Schrödinger equation with external magnetic field. Rostock. Math. Kolloq. 56, 49–54 (2002) · Zbl 1140.35567

[26] Solimini S.: A note on compactness-type properties with respect to Lorentz norms of bounded subsets of a Sobolev space. Ann. Inst. H. Poincaré Anal. Non Linéaire 12, 319–337 (1995) · Zbl 0837.46025

[27] Terracini S.: On positive entire solutions to a class of equations with a singular coefficient and critical exponent. Adv. Differential Equations 1, 241–264 (1996) · Zbl 0847.35045

[28] Fieseler K.-H., Tintarev K.: Concentration Compactness: Functional-Analytic Grounds and Applications. Imperial College Press, London (2007) · Zbl 1118.49001