Cuchí, J. E.; **Gil-Rivero, A.**; **Molina, A.**; **Ruiz, E.**

An approximate global solution of Einstein's equation for a rotating compact source with linear equation of state. (English) Zbl 1271.83027

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Summary: We use analytic perturbation theory to present a new approximate metric for a rigidly rotating perfect fluid source with equation of state (EOS) $\epsilon + (1-n)p = \epsilon_0$. This EOS includes the interesting cases of strange matter, constant density and the fluid of the Wahlquist metric. It is fully matched to its approximate asymptotically flat exterior using Lichnerowicz junction conditions and it is shown to be a totally general matching using Darmois-Israel conditions and properties of the harmonic coordinates. Then we analyse the Petrov type of the interior metric and show first that, in accordance with previous results, in the case corresponding to Wahlquist's metric it can not be matched to the asymptotically flat exterior. Next, that this kind of interior can only be of Petrov types I, D or (in the static case) O and also that the non-static constant density case can only be of type I. Finally, we check that it can not be a source of Kerr's metric.

MSC:

83C25	Approximation procedures, weak fields in general relativity and gravitational theory
83C15	Exact solutions to problems in general relativity and gravitational theory
83C55	Macroscopic interaction of the gravitational field with matter (hydrodynamics, etc.)
85A15	Galactic and stellar structure
83C20	Classes of solutions; algebraically special solutions, metrics with symmetries for problems in general relativity and gravitational theory

Keywords:

perturbation theory; Wahlquist metric; axistationary; rotating stars; spacetime matching; global; approximate; Petrov type; equation of state; EOS; Kerr's metric

Software:

xPerm

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References:

[1] Neugebauer, G; Meinel, R, General relativistic gravitational field of a rigidly rotating disk of dust: solution in terms of ultraelliptic functions, Phys. Rev. Lett., 75, 3046-3047, (1995) · Zbl 1020.83522 · doi:10.1103/PhysRevLett.75.3046

[2] Klein, C, Exact relativistic treatment of stationary counterrotating dust disks: boundary value problems and solutions, Phys. Rev. D, 63, 064033, (2001)· doi:10.1103/PhysRevD.63.064033

[3] Maison, D, Are the stationary, axially symmetric Einstein equations completely integrable?, Phys. Rev. Lett., 41, 521-522, (1978)· doi:10.1103/PhysRevLett.41.521

[4] Maison, D.: On the complete integrability of the stationary, axially symmetric Einstein equations. J. Math. Phys. \textbf{20}, 871 (1979)

[5] Wahlquist, HD, Interior solution for a finite rotating body of perfect fluid, Phys. Rev., 172, 1291-1296, (1968)· doi:10.1103/PhysRev.172.1291

[6] Wahlquist, HD, The problem of exact interior solutions for rotating rigid bodies in general relativity, J. Math. Phys., 33, 304, (1992) · Zbl 0850.70199 · doi:10.1063/1.529965

[7] Chinea, FJ; González-Romero, LM, Interior gravitational field of stationary, axially symmetric perfect fluid in irrotational motion, Class. Quantum Gravit., 7, l99-l102, (1990)· doi:10.1088/0264-9381/7/5/001

[8] Ansorg, M; Gondek-Rosińska, D; Villain, L, On the solution space of differentially rotating neutron stars in general relativity, Mon. Not. R. Astron. Soc., 396, 2359-2366, (2009) · Zbl 1176.62077 · doi:10.1111/j.1365-2966.2009.14904.x

[9] Ansorg, M; Fischer, T; Kleinwächter, A; Meinel, R; Petroff, D; Schöbel, K, Equilibrium configurations of homogeneous fluids in general relativity, Mon. Not. R. Astron. Soc., 355, 682-688, (2004)· doi:10.1111/j.1365-2966.2004.08371.x

[10] Mars, M; Senovilla, JMM, On the construction of global models describing rotating bodies; uniqueness of the exterior gravitational field, Mod. Phys. Lett. A, 13, 1509-1519, (1998)· doi:10.1142/S0217732398001583

[11] Bradley, M; Fodor, G; Marklund, M; Perjés, Z, The wahlquist metric cannot describe an isolated rotating body, Class. Quantum Gravit., 17, 351-360, (2000) · Zbl 0961.83022 · doi:10.1088/0264-9381/17/2/306

[12] Sarnobat, P; Hoenselaers, CA, The wahlquist exterior: second-order analysis, Class. Quantum Gravit., 23, 5603, (2006) · Zbl 1101.83015 · doi:10.1088/0264-9381/23/18/007

[13] Ansorg, M; Kleinwächter, A; Meinel, R, Highly accurate calculation of rotating neutron stars, Astron. Astrophys., 381, l49-l52, (2002) · Zbl 1059.85002 · doi:10.1051/0004-6361:20011643

[14] Ansorg, M; Kleinwächter, A; Meinel, R, Highly accurate calculation of rotating neutron stars. detailed description of the numerical methods, Astron. Astrophys., 405, 711-721, (2003) · Zbl 1059.85002 · doi:10.1051/0004-6361:20030618

[15] Stergioulas, N.: Rotating stars in relativity. http://www.livingreviews.org/lrr-2003-3 Living Reviews in Relativity \textbf{6}, (2003) version: lrr-2003-3 · Zbl 1068.83508

[16] Blanchet, L.: Gravitational radiation from post-Newtonian sources and inspiralling compact binaries. http://www.livingreviews.org/lrr-2006-4 Living Reviews in Relativity \textbf{9}, (2006) version: lrr-2006-4 · Zbl 1316.83004

[17] Futamase, T., Itoh, Y.: The post-Newtonian approximation for relativistic compact binaries. http://www.livingreviews.org/lrr-2007-2 Living Reviews in Relativity \textbf{10} (2007) version: lrr-2007-2 · Zbl 1255.83005

[18] Shibata, M., Taniguchi, K.: Coalescence of black hole-neutron star binaries. http://www.livingreviews.org/lrr-2011-6 Living Reviews in Relativity \textbf{14} (2011) version: lrr-2011-6

[19] Bradley, M., Fodor, G.: Quadrupole moment of slowly rotating fluid balls. Phys. Rev. D \textbf{79}, 044018 (2009). doi:10.1103/PhysRevD.79.04401 · Zbl 0809.53069

[20] Cabezas, JA; Martín, J; Molina, A; Ruiz, E, An approximate global solution of einstein's equations for a rotating finite body, Gen. Relativ. Gravit., 39, 707-736, (2007) · Zbl 1157.83311 · doi:10.1007/s10714-007-0414-6

[21] Nozawa, T; Stergioulas, N; Gourgoulhon, E; Eriguchi, Y, Construction of highly accurate models of rotating neutron stars-comparison of three different numerical schemes, Astron. Astrophys. Suppl. Ser., 132, 431-454, (1998)· doi:10.1051/aas:1998304

[22] Cuchí, J.E., Molina, A., Ruiz, E.: Comparing results for a global metric from analytical perturbation theory and a numerical code (2013). http://arxiv.org/abs/1301.7423, arXiv:1301.7423 [gr-qc] · Zbl 1221.85015

[23] Lattimer, JM, Neutron stars and the dense matter equation of state, Astrophys. Space Sci., 336, 67-74, (2011)· doi:10.1007/s10509-010-0529-1

[24] Teichmüller, C; Fröb, M; Maucher, F, Analytical approximation of the exterior gravitational field of rotating neutron stars, Class. Quantum Gravit., 28, 155015, (2011) · Zbl 1221.85015 · doi:10.1088/0264-9381/28/15/155015

[25] Weber, F, Strange quark matter and compact stars, Prog. Part. Nucl. Phys., 54, 193-288, (2005)· doi:10.1016/j.ppnp.2004.07.001

[26] Weissenborn, S; Sagert, I; Pagliara, G; Hempel, M; Schaffner-Bielich, J, Quark matter in massive compact stars, Astrophys. J., 740, l14, (2011)· doi:10.1088/2041-8205/740/1/L14

[27] Darmois, G.: Mémorial des Sciences Mathématiques, vol. XXV. Gauthier-Villars, Paris (1927). Chapter V · Zbl 0657.53046

[28] Lichnerowicz, A., Darmois, G.: Théories relativistes de la gravitation et de l'électromagnétisme: relativité générale et théories unitaires. Masson (1955)

[29] Senovilla, JMM; Chinea, FJ (ed.); González-Romero, LM (ed.), Stationary and axisymmetric perfect-fluid solutions to einstein's equations, (1993), Berlin

[30] Carminati, J, Type-N, shear-free, perfect-fluid spacetimes with a barotropic equation of state, Gen. Relativ. Gravit., 20, 1239-1248, (1988) · Zbl 0657.53046 · doi:10.1007/BF00756051

[31] Mars, M; Senovilla, JMM, Axial symmetry and conformal Killing vectors, Class. Quant. Grav., 10, 1633, (1993) · Zbl 0809.53069 · doi:10.1088/0264-9381/10/8/020

[32] Carter, B, The commutation property of a stationary, axisymmetric system, Commun. Math. Phys., 17, 233-238, (1970) · Zbl 0194.58701 · doi:10.1007/BF01647092

[33] Carter, B.: Killing horizons and orthogonally transitive groups in space-time. J. Math. Phys. \textbf{10}, 70-81 (1969). doi:10.1063/1.1664763 · Zbl 0165.58902

[34] Papapetrou, A, Champs gravitationnels stationnaires à symetrie axiale, Ann. Inst. Henri Poincaré A, 4, 83-105, (1966) · Zbl 0118.23002

[35] Carter, B.: Black hole equilibrium states. In: DeWitt, C., DeWitt, B. (eds.) Black Holes-Les Astres Occlus, vol. 57, pp. 61-124. Gordon and Breach, New York (1973). Golden Oldie republication in doi:10.1007/s10714-009-0888-5 · Zbl 0961.83022

[36] Kundt, W; Trümper, M, Orthogonal decomposition of axi-symmetric stationary spacetimes, Z. Phys. A, 192, 419-422, (1966)

[37] Boyer, R.H.: Rotating fluid masses in general relativity. In: Proceedings of the Cambridge Philosophical Society, vol. 61 (1965) · Zbl 0127.18202

[38] Cuchí, J.E., Molina, A., Ruiz, E.: Comparison of metrics obtained with analytic perturbation theory and a numerical code. In: Beltrán Jiménez, J., Ruiz Cembranos, J.A., Dobado, A., López Maroto, A., De la Cruz Dombriz, A. (eds.) Towards New Paradigms: Proceeding of the Spanish Relativity Meeting 2011, vol. 1458, pp. 371-374. Am. Inst. Phys. Conf. Proc. (2012) http://arxiv.org/abs/1202.6676, arXiv:1202.6676 [gr-qc] · Zbl 1101.83015

[39] Hartle, JB, Slowly rotating relativistic stars. I. equations of structure, Astrophys. J., 150, 1005, (1967)· doi:10.1086/149400

[40] Thorne, KS, Multipole expansions of gravitational radiation, Rev. Mod. Phys., 52, 299-339, (1980)· doi:10.1103/RevModPhys.52.299

[41] Geroch, R, Multipole moments. II. curved space, J. Math. Phys., 11, 2580-2588, (1970) · Zbl 1107.83312 · doi:10.1063/1.1665427

[42] Hansen, RO, Multipole moments of stationary space-times, J. Math. Phys., 15, 46-52, (1974) · Zbl 1107.83304 · doi:10.1063/1.1666501

[43] Bonnor, WB; Vickers, PA, Junction conditions in general relativity, Gen. Relativ. Gravit., 13, 29-36, (1981)

[44] Mars, M; Mena, FC; Vera, R, Linear perturbations of matched spacetimes: the gauge problem and background symmetries, Class. Quantum Gravit., 24, 3673, (2007) · Zbl 1206.83045 · doi:10.1088/0264-9381/24/14/008

[45] Rendall, AD; Schmidt, BG, Existence and properties of spherically symmetric static fluid bodies with a given equation of state, Class. Quantum Gravit., 8, 985, (1991) · Zbl 0724.53055 · doi:10.1088/0264-9381/8/5/022

[46] Martín, J; Molina, A; Ruiz, E, Can rigidly rotating polytropes be sources of the Kerr metric?, Class. Quantum Gravit., 25, 105019, (2008) · Zbl 1140.83329 · doi:10.1088/0264-9381/25/10/105019

[47] Stephani, H., Kramer, D., MacCallum, M., Hoenselaers, C., Herlt, E.: Exact Solutions of Einstein's Field Equations. Cambridge University Press, Cambridge (2003) · Zbl 1057.83004 · doi:10.1017/CBO9780511535185

[48] Collinson, CD, The uniqueness of the Schwarzschild interior metric, Gen. Relativ. Gravit., 7, 419-422, (1976)

[49] Kramer, D, Rigidly rotating perfect fluids, Astron. Nachr., 307, 309-312, (1986) · Zbl 0603.76127 · doi:10.1002/asna.2113070519

[50] Senovilla, JMM, Stationary axisymmetric perfect-fluid metrics with $q+ 3 p= const$, Phys. Lett. A, 123, 211-214, (1987)· doi:10.1016/0375-9601(87)90062-4

[51] Martín-García, J.M.: xPerm: fast index canonicalization for tensor computer algebra. Comput. Phys. Commun. \textbf{179}, 597-603 (2008). Available in http://www.xact.es · Zbl 0194.58701