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High-order semi-implicit time-integrators for a triangular discontinuous Galerkin oceanic shallow water model. (English) ⃞Zbl 1267.76010

Summary: We extend the explicit in time high-order triangular discontinuous Galerkin (DG) method to semi-implicit (SI) and then apply the algorithm to the two-dimensional oceanic shallow water equations; we implement high-order SI time-integrators using the backward difference formulas from orders one to six. The reason for changing the time-integration method from explicit to SI is that explicit methods require a very small time step in order to maintain stability, especially for high-order DG methods. Changing the time-integration method to SI allows one to circumvent the stability criterion due to the gravity waves, which for most shallow water applications are the fastest waves in the system (the exception being supercritical flow where the Froude number is greater than one). The challenge of constructing a SI method for a DG model is that the DG machinery requires not only the standard finite element-type area integrals, but also the finite volume-type boundary integrals as well. These boundary integrals pose the biggest challenge in a SI discretization because they require the construction of a Riemann solver that is the true linear representation of the nonlinear Riemann problem; if this condition is not satisfied then the resulting numerical method will not be consistent with the continuous equations.

In this paper we couple the SI time-integrators with the DG method while maintaining most of the usual attributes associated with DG methods such as: high-order accuracy (in both space and time), parallel efficiency, excellent stability, and conservation. The only property lost is that of a compact communication stencil typical of time-explicit DG methods; implicit methods will always require a much larger communication stencil. We apply the new high-order SI DG method to the shallow water equations and show results for many standard test cases of oceanic interest such as: standing, Kelvin and Rossby soliton waves, and the Stommel problem.

The results show that the new high-order SI DG model, that has already been shown to yield exponentially convergent solutions in space for smooth problems, results in a more efficient model than its explicit counterpart. Furthermore, for those problems where the spatial resolution is sufficiently high compared with the length scales of the flow, the capacity to use high-order (HO) time-integrators is a necessary complement to the employment of HO space discretizations, since the total numerical error would be otherwise dominated by the time discretization error. In fact, in the limit of increasing spatial resolution, it makes little sense to use HO spatial discretizations coupled with low-order time discretizations.

MSC:

76B07	Free-surface potential flows for incompressible inviscid fluids	Cited in **26** Documents
76M10	Finite element methods applied to problems in fluid mechanics	
76M12	Finite volume methods applied to problems in fluid mechanics	
86-08	Computational methods for problems pertaining to geophysics	
65M60	Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs	

Keywords:

discontinuous Galerkin; explicit; finite element; finite volume; implicit; semi-implicit; shallow water; triangle

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TsunAWI

Full Text: DOI

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