

**Lei, Yutian**

**On the regularity of positive solutions of a class of Choquard type equations.** (English)

Zbl 1267.45010

Math. Z. 273, No. 3-4, 883-905 (2013).

This paper is concerned with positive solutions of a class of Choquard type equations. The author studies regularity for integrable solutions by using regularity lifting lemmas. He also establishes the smoothness of those solutions. Choquard equations are equivalent to integral systems involving the Bessel potential and the Riesz potential. Several integral systems involving the Bessel potential were studied by *L. Ma* and *D. Chen* [Math. Comput. Modelling 49, No. 1-2, 379-385 (2009; Zbl 1165.35372)] and by *F. Ma* et al. [Integral Equations Oper. Theory 69, No. 3, 393-404 (2011; Zbl 1227.45005)]. They also studied the integral system involving the Riesz potential.

Reviewer: [Claudio Cuevas \(Pernambuco\)](#)

**MSC:**

45G05 Singular nonlinear integral equations

Cited in **20** Documents

**Keywords:**

Choquard equation; integral equations; integrability intervals; Hardy-Littlewood-Sobolev inequality; Bessel potential; regularity lifting lemmas; Riesz potential

**Full Text:** [DOI](#)

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