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A minimization method and applications to the study of solitons. (English) Zbl 1263.47084
Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 75, No. 12, 4398-4421 (2012).

In the paper under review, the authors prove a general abstract theorem that allows to derive existence of a class of solitons related to field equations. Precisely, a solitary wave is a solution of a field equation whose energy travels as a localized packet and which preserves this localization in time. A soliton is a solitary wave which exhibits some strong form of stability so that it has a particle-like behavior. The solitons studied in the paper are suitable minimizers of a constrained functional, and these are called hylomorphic solitons. The abstract theory developed is further applied to problems related to the nonlinear Schrödinger and Klein-Gordon equations.

Reviewer: [Dian K. Palagachev \(Bari\)](#)

MSC:

- [47J30](#) Variational methods involving nonlinear operators
- [35J50](#) Variational methods for elliptic systems
- [35Q55](#) NLS equations (nonlinear Schrödinger equations)
- [35Q51](#) Soliton equations
- [37K45](#) Stability problems for infinite-dimensional Hamiltonian and Lagrangian systems
- [47N20](#) Applications of operator theory to differential and integral equations

Cited in **6** Documents

Keywords:

lack of compactness; orbital stability; nonlinear Schrödinger equation; lattice; nonlinear Klein-Gordon equation; solitary waves; hylomorphic solitons; vortices

Full Text: [DOI](#) [arXiv](#)

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