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Weak convergence of spectral shift functions for one-dimensional Schrödinger operators.

(English) Zbl 1263.47026
Math. Nachr. 285, No. 14-15, 1799-1838 (2012).

The authors study the manner in which spectral shift functions associated with self-adjoint one-dimensional Schrödinger operators on the finite interval $(0, R)$ converge in the infinite volume limit $R \to \infty$ to the half-line spectral shift function. Relying on a Fredholm determinant approach combined with certain measure theoretic facts, the authors show that prior vague convergence results in the literature in the special case of Dirichlet boundary conditions extend to the notion of weak convergence and arbitrary separated self-adjoint boundary conditions at $x = 0$ and $x = R$.

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MSC:

47B25 Linear symmetric and selfadjoint operators (unbounded)
34L05 General spectral theory of ordinary differential operators
34L25 Scattering theory, inverse scattering involving ordinary differential operators
34L40 Particular ordinary differential operators (Dirac, one-dimensional Schrödinger, etc.)
34B24 Sturm-Liouville theory
34B27 Green's functions for ordinary differential equations
47E05 General theory of ordinary differential operators

Keywords:

spectral shift functions; Fredholm determinants; one-dimensional Schrödinger operators

Full Text: DOI arXiv

References:

[1] Abramowitz, Handbook of Mathematical Functions (1972)

[2] Aizenman, rownian motion and Harnack inequality for Schrödinger operators, Commun. Pure App l. Math 35 pp 209– (1982) · Zbl 0459.60069 · doi:10.1002/cpa.3160350206

[3] I, Theory of Linear Operators in Hilbert Space (1981)

[4] B, Regular approximations of singular Sturm-Liouville problems, Results. Math 23 pp 3– (1993) · Zbl 0778.34015 · doi:10.1007/BF03323127

[5] Bauer, Measure and Integration Theory, de Gruyter Studies in Mathematics (2001) · Zbl 0985.28001 · doi:10.1515/9783110866209

[6] Birman, The spectral shift function. The work of M. G. Krein and its further development, St. Petersbg. Math. J 4 pp 833– (1993)

[7] V. Borovyk Box approximation and related techniques in spectral theory 2008

[8] Borovyk, On the weak and ergodic limit of the spectral shift function, Lett. Math. Phys 100 pp 1– (2012) · Zbl 1243.81077 · doi:10.1007/s11005-011-0524-7

[9] S, Formulas for traces for a singular Sturm-Liouville differential operator, Sov. Math. Dokl 1 pp 451– (1960) · Zbl 0129.06501

[10] Chadan, Inverse Problems in Quantum Scattering Theory (1989) · Zbl 0681.35088 · doi:10.1007/978-3-642-83317-5

[11] M, The Wegner estimate and the integrated density of states for some random operators, Pro c. Indian Acad. Sci. (Math. Sci.) 112 pp 31– (2002) · Zbl 1009.47060 · doi:10.1007/BF02829639

[12] M, The Lp-theory of the spectral shift function, the Wegner estimate, and the integrated density of states for some random operaors, Commun. Math. Phys 218 pp 113– (2001) · Zbl 1042.82024 · doi:10.1007/PL00005555

[13] L, Classical Potential Theory and Its Probabilistic Counterpart (1984) · Zbl 0549.31001

[14] D, An expression for the trace of the difference between two singular differential operators of the Sturm-Liouville type, Dokl. Akad. Nauk SSSR n. Ser 115 pp 878– (1957) · Zbl 0143.31403

[15] B, Real Analysis, Modern Techniques and their Applications (1999) · Zbl 0924.28001

[16] Geisler, Concavity properties of Krein's spectral shift function, Rev. Math. Phys 7 pp 161– (1995) · Zbl 0836.47049 · doi:10.1142/S0129055X95000098

[17] Gesztesy, Nonselfadjoint operators, infinite determinants, and some applications, Russ. J. Math. Phys 12 pp 443– (2005) · Zbl 1201.47028

[18] Gesztesy, (Modified) Fredholm determinants for operators with matrix-valued semi-separable integral kernels revisited, Integral Equations Oper. Theory 47 pp 457– (2003) · Zbl 1084.47015 · doi:10.1007/s00020-003-1170-y

[19] F. Gesztesy R. Nichols

[20] Gesztesy, Symmetrized perturbation determinants and applications to boundary data maps and Krein-type resolvent formulas, Proc, London Math. Soc 3 pp 577– (2012) · Zbl 1247.34030 · doi:10.1112/plms/pdr024

[21] C, Introduction to the Theory of Linear Nonselfadjoint Operators (1969)

[22] I. S. S I. M. M 1980

[23] Rümm, Two theorems about \documentclass{article}\usepackage{amssymb}\begin{document}\pagestyle{empty}\({\mathcal C}_p\)\end{document}, Rep. Math. Phys 4 pp 211– (1973) · Zbl 0258.47022 · doi:10.1016/0034-4877(73)90026-8

[24] D, The integrated density of states for some random operators with nonsign definite otentials, J. Funct. Anal 195 pp 12– (2002) · Zbl 1013.60046 · doi:10.1006/jfan.2002.3947

[25] D, The spectral shift function for compactly supported perturbations of Schrödinger operators on large bounded domains, Proc. Am. Math. Soc 138 pp 2141– (2010) · Zbl 1190.81127 · doi:10.1090/S0002-9939-10-10264-0

[26] P. D. D P. Müller Uniform convergence of spectral shift functions

[27] Hundertmark, Bounds on the spectral shift function and the density of states, Commun. Math. Phys 262 pp 489– (2006) · Zbl 1121.47006 · doi:10.1007/s00220-005-1460-0

[28] Hundertmark, An optimal Lp-bound on the Krein spectral shift function, J. Anal. Math 87 pp 199– (2002) · Zbl 1054.47020 · doi:10.1007/BF02868474

[29] Hupfer, Existence and uniqueness of the integrated density of states for Schrödinger operators with magnetic fields and unbounded random potentials, Rev. Math. Phys 13 pp 1547– (2001) · Zbl 1029.81027 · doi:10.1142/S0129055X01001083

[30] Jost, On the scattering of a particle by a static potential, Phys. Rev 82 pp 840– (1951) · Zbl 0042.45206 · doi:10.1103/PhysRev.82.840

[31] Kato, Perturbation Theory for Linear Operators (1980) · Zbl 0435.47001

[32] Kirsch, Small perturbations and the eigenvalues of the laplacian on large bounded domains, Proc. Am. Math. Soc 101 pp 509– (1987) · Zbl 0643.35074 · doi:10.1090/S0002-9939-1987-0908658-9

[33] V. Kostrykin Die spektrale Shiftfunction und ihre Anwendungen auf zufällige Schrödingeroperatoren 1999

[34] Kostrykin, Scattering theory approach to random Schrödinger operators in one dimension, Rev. Math. Phys 11 pp 187– (1999) · Zbl 0957.60101 · doi:10.1142/S0129055X99000088

[35] Kostrykin, The density of states and the spectral shift density of random Schrödinger operators, Rev. Math. Phys 12 pp 807– (2000) · Zbl 0961.47022 · doi:10.1142/S0129055X00000320

[36] Krasnoselskii, Integral Operators in Spaces of Summable Functions (1976)· doi:10.1007/978-94-010-1542-4

[37] Krüger, Relative oscillation theory for Sturm-Liouville operators extended, J. Funct. Anal 254 pp 1702– (2008) · Zbl 1144.34014 · doi:10.1016/j.jfa.2007.10.007

[38] Krüger, Relative oscillation theory, weighted zeros of the Wronskian, and the spectral shift function, Commun. Math. Phys 287 pp 613– (2009) · Zbl 1186.47009 · doi:10.1007/s00220-008-0600-8

[39] Y. Latushkin A. Sukhtayev 5 939 970 2012

[40] M, Introduction to Spectral Theory: Selfadjoint Ordinary Differential Operators, Transl. Math (1975)

[41] Nakamura, A remark on the Dirichlet-Neumann decoupling and the integrated density of states, J. Funct. Anal 179 pp 136– (2001) · Zbl 0970.35084 · doi:10.1006/jfan.2000.3683

[42] Reed, Methods of Modern Mathematical Physics. IV: Analysis of Operators (1978) · Zbl 0401.47001

[43] Reed, Methods of Modern Mathematical Physics. I: Functional Analysis, revised and enlarged edition (1980)

[44] Schechter, Operator Methods in Quantum Mechanics (1981)

[45] Simon, Notes on infinite determinants of Hilbert space operators, Adv. Math 24 pp 244– (1977) · Zbl 0353.47008 · doi:10.1016/0001-8708(77)90057-3

[46] Simon, Schrödinger semigroups, Bull. Am. Math. Soc 7 pp 447– (1982) · Zbl 0524.35002 · doi:10.1090/S0273-0979-1982-15041-8

[47] Simon, Trace Ideals and Their Applications, Mathematical Surveys and Monographs (2005) · Zbl 1074.47001

[48] V, Efficient bounds for the spectral shift function, Ann. Inst. Henri Poincaré 58 pp 55– (1993)

[49] Stolz, Approximation of isolated eigenvalues of ordinary differential operators, J. Reine Angew. Math 445 pp 31– (1993) · Zbl 0781.34052

[50] Stolz, Approximation of isolated eigenvalues of general singular ordinary differential operators, Results. Math 28 pp 345– (1995) · Zbl 0842.47029 · doi:10.1007/BF03322261

[51] Teschl, On the approximation of isolated eigenvalues of ordinary differential operators, Proc. Am. Math. Soc 136 pp 2473– (2008) · Zbl 1156.34076 · doi:10.1090/S0002-9939-08-09140-5

[52] Weidmann, Linear Operators in Hilbert Spaces, Graduate Texts in Mathematics (1980) · Zbl 0434.47001 · doi:10.1007/978-

1-4612-6027-1

[53] R, Mathematical Scattering Theory, General Theory, Transl. Math. Monographs (1992)

[54] R, Perturbation determinants, the spectral shift function, trace identities, and all that, Funct. Anal. Appl 41 pp 217– (2007) · Zbl 1222.47072 · doi:10.1007/s10688-007-0019-6

[55] R, Mathematical Scattering Theory, Analytic Theory, Math. Surveys and Monographs (2010)