Cortissoz, Jean C.

On the blow-up behavior of a nonlinear parabolic equation with periodic boundary conditions. (English) Zbl 1263.35047

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The author considers the blow-up behavior of real solutions in the d-dimensional unit cube to the quasilinear problem

$$\begin{cases} u_t = u\Delta u + u^2 & \text{on } [0,1]^d \times (0,T), \\ u(\cdot,0) = \psi & \text{in } [0,1]^d, \end{cases} \tag{1}$$

with periodic boundary conditions. Consider the family of seminorms for $v \in L^2([0,1]^d)$ given by

$$\|v\|_{f(s),\beta} := \sup_{\xi\in\mathbb{Z}^d,\xi\neq 0} |\beta|^\beta |f(\xi)|\,|\hat{v}(\xi)|,$$

where $\hat{v}$ denotes the Fourier transform of v. The following result is proved: put $\alpha(1) := 3/2$ and $\alpha(d) := d$ if $d \geq 2$. There are positive constants c_d and C_k, $k \in \mathbb{N}$, such that if $\psi \in L^2([0,1]^d)$ satisfies

$$\int_{[0,1]^d} \psi \geq c_d \|\psi\|_{\log^{\frac{3}{2}}(|s|+2),\alpha(d)},$$

then the Cauchy problem (1) has a unique solution u with initial condition ψ. This solution blows up at a time $T \in (0,\infty)$ and satisfies

$$\left\| u(x,t) - \int_{[0,1]^d} u(x,t)\,dx \right\|_{C^k([0,1]^d)} < C_k(T-t) \qquad \text{for all } t \in [0,T),\ k \in \mathbb{N}.$$

In other words, its limiting profile is flat.

The proof rests on Fourier transforms and Galerkin approximations for solutions of (1).

Reviewer: Nils Ackermann (Mexico City)

MSC:

35B44 Blow-up in context of PDEs
35K59 Quasilinear parabolic equations
35K20 Initial-boundary value problems for second-order parabolic equations
35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs

Keywords:

quasilinear parabolic equation; blow-up solution; asymptotic profile; Fourier series; flat limiting profile; Fourier transform

Full Text: DOI

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