Földes, Juraj; Poláčik, P.

Convergence to a steady state for asymptotically autonomous semilinear heat equations on $\mathbb{R}^N$. (English) Zbl 1263.35035

J. Differ. Equations 251, No. 7, 1903-1922 (2011).

The authors treat semilinear parabolic equations of the form

$$u_t = \Delta u + f(u) + h(x,t), \qquad (x,t) \in \mathbb{R}^N \times (0,\infty), \tag{1}$$

that are asymptotically autonomous in the sense that $\|h(\cdot,t)\|_\infty \to 0$ as $t \to \infty$. Here f is assumed to be continuously differentiable and to satisfy $f(0) = 0$ and $f'(0) < 0$. Consider a global, bounded and nonnegative solution u of (1) that decays in x, uniformly in t:

$$\lim_{|x|\to\infty} \sup_{t>0} u(x,t) = 0.$$

Define its ω-limit set in a suitable Banach space of states by

$$\omega(u) := \{v : u(\cdot,t_k) \to v \text{ for some sequence } t_k \to \infty\}.$$

A positive solution of $\Delta u + f(u) = 0$ on $\mathbb{R}^N$ that decays to 0 as $|x| \to \infty$ is called a *ground state*. A ground state is always radially symmetric about some point and radially decaying.

Under these assumptions it is shown that either $\omega(u) = \{0\}$ or that $\omega(u)$ consists entirely of ground states. If in addition $h(\cdot,t)$ decays exponentially in a suitable Hölder class, then either $\omega(u) = \{0\}$ or $\omega(u)$ consists of exactly one ground state. Since

$$\lim_{t\to\infty} \operatorname{dist}_\infty(u(\cdot,t), \omega(u)) = 0,$$

these results yield, respectively, quasiconvergence and convergence of u in the L^∞-sense.

Reviewer: Nils Ackermann (Mexico City)

MSC:

35B40 Asymptotic behavior of solutions to PDEs
35K58 Semilinear parabolic equations
35B09 Positive solutions to PDEs
35B07 Axially symmetric solutions to PDEs

Keywords:

quasiconvergence; ground state; radial symmetry

Full Text: DOI

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