Poláčik, P.

Threshold solutions and sharp transitions for nonautonomous parabolic equations on $\mathbb{R}^N$.

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Author's abstract: "This paper is devoted to a class of nonautonomous parabolic equations of the form $u_t = \Delta u + f(t,u)$ on $\mathbb{R}^N$. We consider a monotone one-parameter family of initial data with compact support, such that for small values of the parameter the corresponding solutions decay to zero, whereas for large values they exhibit a different behavior (either blowup in finite time or locally uniform convergence to a positive constant steady state). We are interested in the set of intermediate values of the parameter for which neither of these behaviors occurs. We refer to such values as threshold values and to the corresponding solutions as threshold solutions. We prove that the transition from decay to the other behavior is sharp: there is just one threshold value. We also describe the behavior of the threshold solution: it is global, bounded, and asymptotically symmetric in the sense that all its limit profiles, as $t \to \infty$, are radially symmetric about the same center. Our proofs rely on parabolic Liouville theorems, asymptotic symmetry results for nonlinear parabolic equations, and theorems on exponential separation and principal Floquet bundles for linear parabolic equations."

Reviewer: Nils Ackermann (Mexico City)

MSC:

35K58 Semilinear parabolic equations
35B40 Asymptotic behavior of solutions to PDEs
35B07 Axially symmetric solutions to PDEs
35B53 Liouville theorems and Phragmén-Lindelöf theorems in context of PDEs

Keywords:
asymptotic symmetry; parabolic Liouville theorems; principal Floquet bundles

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