Carette, Mathieu; Francaviglia, Stefano; Kapovich, Ilya; Martino, Armando
Spectral rigidity of automorphic orbits in free groups. (English) Zbl 1261.20040
Algebr. Geom. Topol. 12, No. 3, 1457-1486 (2012); corrigendum 14, No. 5, 3081-3088 (2014).

Let F_N be the finitely generated free group of rank $N \geq 2$. The non-projectivized Outer space cv_N is introduced by *M. Culler* and *K. Vogtmann* [in Invent. Math. 84, 91-119 (1986; Zbl 0589.20022)]. A basic fact in the theory of Outer space states that every $T \in \mathrm{cv}_N$ is uniquely determined by its translation length function $\|\cdot\|: F_N \to \mathbb{R}$, where for every $g \in F_N$ $\|g\|_T = \min_{x\in T} d_T(x, gx)$ is the translation length of g (namely the Marked Length Rigidity Conjecture holds for Outer space).

A subset $R \subseteq F_N$ is called spectrally rigid if whenever $T_1, T_2 \in \mathrm{cv}_N$ are such that $\|g\|_{T_1} = \|g\|_{T_2}$ for every $g \in R$, then $T_1 = T_2$ in cv_N. As noted above $R = F_N$ is spectrally rigid.

It is proved by *J. Smillie* and *K. Vogtmann* [Mich. Math. J. 39, No. 3, 485-493 (1992; Zbl 0773.05058)] for $N \geq 3$ and by *M. M. Cohen, M. Lustig* and *M. Steiner* [Publ., Math. Sci. Res. Inst. 19, 183-187 (1991; Zbl 0826.20028)] for $N = 2$, that there does not exist a finite spectrally rigid subset of F_N. *I. Kapovich* [Proc. Am. Math. Soc. 140, No. 5, 1549-1560 (2012; Zbl 1268.20043)] has given a class of examples of spectrally rigid subsets of a free group.

In paper under consideration the authors obtain a very different class of examples of spectrally rigid subsets of free groups.

Their main result is: Theorem A. Let $N \geq 2$ and let $H \leq \mathrm{Aut}(F_N)$ be an ample subgroup. Let $g \in F_N$ be an arbitrary nontrivial element; in the case $N = 2$ we also assume that $g \in F_2 = F(a, b)$ is not conjugate to a nonzero power of $[a, b]$ in F_2. Then the orbit $Hg = \{\varphi(g) : \varphi \in H\}$ is a spectrally rigid subset of F_N.

Here the subgroup H is ample means that the image of H in $\mathrm{Out}(F_N)$ contains an infinite normal subgroup of $\mathrm{Out}(F_N)$.

For the proof of this theorem the authors firstly prove that the set $\mathcal{P}_N$ of all the primitive elements in F_N is a spectrally rigid subset in F_N (Theorem 3.4 in the paper).

After that the authors use heavily the machinery of geodesic currents on free groups, and particularly exploit the geometric intersection form between trees and currents, constructed by *I. Kapovich* [Contemp. Math. 394, 149-176 (2006; Zbl 1110.20034)] and *I. Kapovich* and *M. Lustig* [Geom. Topol. 13, No. 3, 1805-1833 (2009; Zbl 1194.20046)].

The arguments for the proof of Theorem 3.4 are derived from *S. Francaviglia* and *A. Martino* [Publ. Mat., Barc. 55, No. 2, 443-473 (2011; Zbl 1268.20042)], these arguments give the following 'relative rigidity' result:

Theorem B. Let $T \in \mathrm{cv}_N$ be arbitrary. There exists a finite set S (depending on T) of primitive elements in F_N with the following property: Whenever $T' \in \mathrm{cv}_N$ is such that $\|g\|_{T'} = \|g\|_T$ for every $g \in S$ then $T = T'$ in cv_N.

Theorem A applies to the cases where $H = \mathrm{Aut}(F_N)$ $(N \geq 2)$ or where $H \leq \mathrm{Aut}(F_N)$ $(N \geq 3)$ is the kernel of the natural homomorphism from $\mathrm{Aut}(F_N)$ to $\mathrm{Aut}(F_N/\gamma_2(F_N))$. This theorem also implies that for $N \geq 3$ any $\mathrm{Aut}(F_N)$-invariant subset of F_N with more than one element is spectrally rigid in F_N.

The paper concludes with the discussion of several open problems motivated by the results of this paper.

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MSC:

20F65	Geometric group theory
20E05	Free nonabelian groups
20E36	Automorphisms of infinite groups
20F05	Generators, relations, and presentations of groups
20E08	Groups acting on trees
57M07	Topological methods in group theory
57M50	General geometric structures on low-dimensional manifolds
53C24	Rigidity results

Keywords:

marked length spectrum; spectral rigidity; free groups; outer space; translation length functions; spectrally rigid subsets

Full Text: DOI

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