Gubarev, V. Yu.

The subspace $L((x_1 \wedge \ldots \wedge x_k)^m)$ of $S^m(\wedge^k \mathbb{R}^n)$. (English. Russian original) Zbl 1257.15015

Algebra Logic 49, No. 4, 305-325 (2010); translation from Algebra Logika 49, No. 4, 451-478 (2010).

Summary: Let $\wedge^k\mathbb{R}^n$ be the kth outer power of a space $\mathbb{R}^n$, $V(m,n,k) = S^m(\wedge^k\mathbb{R}^n)$ the mth symmetric power of $\mathbb{R}^n$, and $V_0 = L((x_1) \wedge \ldots \wedge x_k)^m : x_i \in \mathbb{R}^n)$. We construct a basis and compute a dimension of V_0 for $m = 2$, and for m arbitrary, present an effective algorithm of finding a basis and computing a dimension for the space $V_0(m,n,k)$. An upper bound for the dimension of V_0 is established, which implies that $\lim_{m\to\infty} \frac{\dim V_0(m,n,k)}{\dim V(m,n,k)} = 0$. The obtained results are applied to study a Grassmann variety and finite-dimensional Lie algebras.

MSC:

15A75 Exterior algebra, Grassmann algebras
14M15 Grassmannians, Schubert varieties, flag manifolds

Keywords:

symmetric power of space; outer power of space; Grassmann variety; algorithm; dimension; Lie algebra

Full Text: DOI

References:

[1] V. A. Sharafutdinov, Integral Geometry of Tensor Fields [in Russian], Nauka, Novosibirsk (1993). · Zbl 0795.53070

[2] V. Sharafutdinov, "Slice-by-slice reconstruction algorithm for vector tomoghraphy with incomplete data," Inv. Probl., 23, No. 6, 2603–2627 (2007). · Zbl 1135.65411 · doi:10.1088/0266-5611/23/6/021

[3] V. Yu. Gubarev, "On the subspace L((x y) m) of \$\$ \{S\^m\}\(\backslash\)left(\{\{ \(\backslash\)wedge\^2\}\{\(\backslash\)mathbb\{R\} \(\backslash\)right) \$\$," Sib. Mat. Zh., 50, No. 3, 503–514 (2009). · Zbl 1224.15051 · doi:10.1007/s11202-009-0046-z

[4] A. I. Kostrikin and Yu. I. Manin, Linear Algebra and Geometry [in Russian], MGU, Moscow (1980).

[5] J. A. MacDougall, "A survey of length problems in Grassmann spaces," in Algebraic Structures and Applications, Lect. Notes Pure Appl. Math., 74, Marcel Dekker (1982), pp. 133–148. · Zbl 0481.15015

[6] É. B. Vinberg, A Course in Algebra [in Russian], Factorial (2001).

[7] J. Riordan, Combinatorial Identities, Wiley, New York (1968). · Zbl 0194.00502

[8] S. R. Ghorpade, A. R. Ratil, and H. K. Pillai, "Decomposable subspaces, linear sections of Grassmann varieties, and higher weights of Grassmann codes," Finite Fields Appl., 15, No. 1, 54–68 (2009). · Zbl 1155.14019 · doi:10.1016/j.ffa.2008.08.001

[9] R. Westwick, "Linear transformations on Grassmann spaces," Pac. J. Math., 14, 1123–1127 (1964). · Zbl 0136.16402 · doi:10.2140/pjm.1964.14.1123

[10] L. J. Cummings, "Decomposable symmetric tensors," Pac. J. Math., 35, 65–77 (1970). · Zbl 0202.03702 · doi:10.2140/pjm.1970.35.65