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A convex polynomial that is not sos-convex. (English) Zbl 1254.90159

Summary: A multivariate polynomial \( p(x) = p(x_1, \ldots, x_n) \) is sos-convex if its Hessian \( H(x) \) can be factored as \( H(x) = M^T(x)M(x) \) with a possibly nonsquare polynomial matrix \( M(x) \). It is easy to see that sos-convexity is a sufficient condition for convexity of \( p(x) \). Moreover, the problem of deciding sos-convexity of a polynomial can be cast as the feasibility of a semidefinite program, which can be solved efficiently. Motivated by this computational tractability, it is natural to study whether sos-convexity is also a necessary condition for convexity of polynomials. In this paper, we give a negative answer to this question by presenting an explicit example of a trivariate homogeneous polynomial of degree eight that is convex but not sos-convex.

MSC:
90C25 Convex programming
52A41 Convex functions and convex programs in convex geometry
90C22 Semidefinite programming

Keywords:
convexity; sos-convexity; sum of squares; semidefinite programming

Software:
SOS.m2; Sostools; YALMIP; SeDuMi; Macaulay2

Full Text: DOI arXiv

References:


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