A uniform contraction principle for bounded apollonian embeddings. (English)


Summary: Let \( \hat{H} = H \cup \{\infty\} \) denote the standard one-point completion of a real Hilbert space \( H \).

Given any non-trivial proper subset \( U \subset \hat{H} \) one may define the so-called “Apollonian” metric \( d_U \) on \( U \). When \( U \subset V \subset \hat{H} \) are nested proper subsets, we show that their associated Apollonian metrics satisfy the following uniform contraction principle: Let \( \Delta = \text{diam}_V(U) \in [0, +\infty) \) be the diameter of the smaller subset with respect to the large. Then for every \( x, y \in U \) we have

\[
\Delta d_V(x, y) \leq \tanh \frac{\Delta}{4} d_U(x, y).
\]

In dimension one, this contraction principle was established by G. Birkhoff [Trans. Am. Math. Soc. 85, 219–227 (1957; Zbl 0079.13502)] for the Hilbert metric of finite segments on \( \mathbb{RP}^1 \). In dimension two it was shown by L. Dubois [J. Lond. Math. Soc., II. Ser. 79, No. 3, 719–737 (2009; Zbl 1172.15011)] for subsets of the Riemann sphere \( \hat{C} \sim \mathbb{R}^2 \). It is new in the generality stated here.

MSC:

- 30F45 Conformal metrics (hyperbolic, Poincaré, distance functions)
- 53A30 Conformal differential geometry (MSC2010)
- 47H09 Contraction-type mappings, nonexpansive mappings, A-proper mappings, etc.
- 30C35 General theory of conformal mappings

Keywords:

- one-point completion of Hilbert spaces; Apollonian metric

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References:


