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Estimates for blow-up solutions to nonlinear elliptic equations with p-growth in the gradient.

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For $N \geq 2$ and a bounded domain $\Omega \subseteq \mathbb{R}^N$ denote the symmetrization of Ω by $\Omega^\#$, the open ball in $\mathbb{R}^N$ such that $|\Omega| = |\Omega^\#|$. Denote by Δ_p for $p > 1$ the usual p-Laplacian and consider the problems

$$\begin{cases} \Delta_p u \pm |\nabla u|^p = f(u), & \text{in } \Omega, \\ u(x) \to \infty, & \text{as } x \to \partial\Omega, \end{cases} \tag{1}$$

and

$$\begin{cases} \Delta_p u \pm |\nabla u|^p = f(u), & \text{in } \Omega^\#, \\ u(x) \to \infty, & \text{as } x \to \partial\Omega^\#. \end{cases} \tag{2}$$

Solutions of this kind are commonly called large solutions.

In case of the plus sign in front of the gradient term, it is assumed that $\beta(s) := (p-1)^{1-p} s^{p-1} f((p-1)\log s)$ is continuous, increasing, satisfies $\beta(0) = 0$ and Keller's condition, which is typical for the existence theory of large solutions of a related transformed problem. On the other hand, in the case of a negative sign in front of the gradient term, assume that $F(r) := (p-1)^{1-p} r^{p-1} f((1-p)\log r)$ is decreasing and satisfies $\lim_{r\to 0+} F(r) < +\infty$.

It is proved that if u is a weak solution of (1) and v the unique radial solution of (2) then

$$\operatorname{ess\,inf}_{x\in\Omega} u(x) \geq \operatorname{ess\,inf}_{x\in\Omega^\#} v(x).$$

The results are formulated in much more generality, allowing for general differential operators in (1) that satisfy certain growth conditions related to the operators in (2). The positive case is proved using the radial rearrangement of the solution, and the proof for the negative case involves the maximum principle.

Reviewer: Nils Ackermann (Mexico City)

MSC:

35J92 Quasilinear elliptic equations with p-Laplacian
35J62 Quasilinear elliptic equations
35J25 Boundary value problems for second-order elliptic equations
35B06 Symmetries, invariants, etc. in context of PDEs
35B09 Positive solutions to PDEs
35B44 Blow-up in context of PDEs

Keywords:

p-Laplacian equations; blow-up solutions; large solutions; rearrangements

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