Wang, Qiang

On generalized Lucas sequences. (English) [Zbl 1246.11039]


For a fixed primitive $(4k+2)$-th root of unity $\eta$, the unsigned generalized Lucas sequence of order $k$ is

$$a_n = \sum_{t=1 \atop t \text{ odd}}^{2k} (\eta^t + \eta^{-t})^n.$$ 

When $k = 2$ this gives the classical Lucas sequence. The signed generalized Lucas sequence of order $k$ is the same sum but over even $t$.

The author gives formulas for the characteristic polynomials $g_k(x)$ and $f_k(x)$, respectively, of these integer sequences. He notes that $f_k(x)g_k(x) = E_{2k}(x)$, the Dickson polynomial of the second kind, and uses this to obtain factorizations of $f_k$ and $g_k$. He further gets explicit expressions for the remainder of Dickson polynomials of the first kind divided by $g_k$. This in turn leads to a characterization of permutation binomials of the form $x^r(x^{ea} + 1)$.

For the entire collection see [Zbl 1202.05003].

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11B39 Fibonacci and Lucas numbers and polynomials and generalizations
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Lucas sequences; finite fields; Dickson polynomials; permutation polynomials

Software:
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