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Small solutions of nonlinear Schrödinger equations near first excited states. (English)

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Summary: Consider a nonlinear Schrödinger equation in \mathbb{R}^3 whose linear part has three or more eigenvalues satisfying some resonance conditions. Solutions which are initially small in $H^1 \cap L^1(\mathbb{R}^3)$ and inside a neighborhood of the first excited state family are shown to converge to either a first excited state or a ground state at time infinity. An essential part of our analysis is on the linear and nonlinear estimates near nonlinear excited states, around which the linearized operators have eigenvalues with nonzero real parts and their corresponding eigenfunctions are not uniformly localized in space.

MSC:

[35Q55](#) NLS equations (nonlinear Schrödinger equations)

[81Q05](#) Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics

[35P05](#) General topics in linear spectral theory for PDEs

Cited in **6** Documents

Keywords:

[nonlinear Schrödinger equation](#); [first excited state](#)

Full Text: [DOI](#) [arXiv](#)

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