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**An explicit Chebyshev pseudospectral multigrid method for incompressible Navier-Stokes equations.** (English) [Zbl 1242.76224](#)

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Summary: The two-dimensional steady incompressible Navier-Stokes equations in the form of primitive variables have been solved by Chebyshev pseudospectral method. The pressure and velocities are coupled by artificial compressibility method and the NS equations are solved by pseudotime method with an explicit four-step Runge-Kutta integrator. In order to reduce the computational time cost, we propose the spectral multigrid algorithm in full approximation storage (FAS) scheme and implement it through V-cycle multigrid and full multigrid (FMG) strategies. Four iterative methods are designed including the single grid method; the full single grid method; the V-cycle multigrid method and the FMG method. The accuracy and efficiency of the numerical methods are validated by three test problems: the modified one-dimensional Burgers equation; the Taylor vortices and the two-dimensional lid driven cavity flow. The computational results fit well with the exact or benchmark solutions. The spectral accuracy can be maintained by the single grid method as well as the multigrid ones, while the time cost is greatly reduced by the latter. For the lid driven cavity flow problem, the FMG is proved to be the most efficient one among the four iterative methods. A speedup of nearly two orders of magnitude can be achieved by the three-level multigrid method and at least one order of magnitude by the two-level multigrid method.

**MSC:**

**76M22** Spectral methods applied to problems in fluid mechanics

**76D05** Navier-Stokes equations for incompressible viscous fluids

Cited in **8** Documents

**Software:**

[Matlab](#)

**Full Text:** [DOI](#)

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