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A discontinuous enrichment method for variable-coefficient advection-diffusion at high Péclet number. (English) Zbl 1242.76125

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Summary: A discontinuous Galerkin method with Lagrange multipliers is presented for the solution of variable-coefficient advection-diffusion problems at high Péclet number. In this method, the standard finite element polynomial approximation is enriched within each element with free-space solutions of a local, constant-coefficient, homogeneous counterpart of the governing partial differential equation. Hence in the two-dimensional case, the enrichment functions are exponentials, each exhibiting a sharp gradient in a carefully chosen flow direction. The continuity of the enriched approximation across the element interfaces is enforced weakly by the aforementioned Lagrange multipliers. Numerical results obtained for two benchmark problems demonstrate that elements based on the proposed discretization method are far more competitive for variable-coefficient advection-diffusion analysis in the high Péclet number regime than their standard Galerkin and stabilized finite element comparables.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
76R50 Diffusion
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

Keywords:

advection-diffusion; discontinuous enrichment method; discontinuous Galerkin method; high Péclet number; Lagrange multipliers

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