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**An interplay between Jensen's and Pexider's functional equations on semigroups.** (English)

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Summary: Let  $(S, +)$  and  $(G, +)$  be two commutative semigroups. Assuming that the latter one is cancellative, we deal with functions  $f : S \rightarrow G$  satisfying the Jensen functional equation written in the form

$$2f(x + y) = f(2x) + f(2y).$$

It turns out that three functions  $f, g, h : S \rightarrow G$  satisfying the functional equation of Pexider

$$f(x + y) = g(x) + h(y)$$

must necessarily be Jensen. The validity of the converse implication is also studied with emphasis placed on a very special Pexider equation

$$\varphi(x + y) + \delta = \varphi(x) + \varphi(y),$$

where  $\delta$  is a fixed element of  $G$ . Plainly, the main goal is to express the solutions of both: Jensen and Pexider equations in terms of semigroup homomorphisms.

Bearing in mind the algebraic nature of the functional equations considered, we were able to establish our results staying away from topological tools.

**MSC:**

39B52 Functional equations for functions with more general domains and/or ranges

20M14 Commutative semigroups

Cited in 2 Documents

**Keywords:**

Jensen's functional equation; Pexider's functional equations; commutative semigroups; semigroup homomorphisms