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Construction of optimal cubature formulas related to computer tomography. (English)

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Summary: We study the problem of the optimization of approximate integration on the class of functions defined on the parallelepiped $\Pi_d = [0, a_1] \times \cdots \times [0, a_d], a_1, \cdots, a_d > 0$, having a given majorant for the modulus of continuity (relative to the l_1-metric in $\mathbb{R}^d$). An optimal cubature formula, which uses as information integrals of f along intersections of Π_d with n arbitrary $(d-1)$-dimensional hyperplanes in $\mathbb{R}^d (d > 1)$ is obtained. We also find an asymptotically optimal sequence of cubature formulas, whose information functionals are integrals of f along intersections of Π_d with shifts of $(d-2)$-dimensional coordinate subspaces of $\mathbb{R}^d (d > 2)$.

MSC:

65D30 Numerical integration
65D32 Numerical quadrature and cubature formulas
41A55 Approximate quadratures
41A63 Multidimensional problems

Keywords:

optimal cubature formula; modulus of continuity; Radon transform

Full Text: DOI

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