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A quasi-analytical shock solution for general nonlinear progressive waves. (English)

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Summary: Many physical phenomena are concerned with the propagation of weak nonlinear waves that can be modeled under the form of a generalized Burgers equation. Physical examples include nonlinearities that can be either quadratic (nonlinear acoustical waves in fluids or longitudinal waves in solids), cubic (nonlinear shear waves in isotropic soft solids), or nonpolynomial (Buckley-Leverett equation for diphasic fluids, models for car traffic, Hertz contact in granular media). A new weak shock formulation of the generalized Burgers equation using an intermediate variable called "potential" is proposed. This formulation is a generalization to nonquadratic nonlinearities of the method originally proposed by *J.M. Burgers* himself [Nederl. Akad. Wet., Proc., Ser. B 57, 159–169 (1954; Zbl 0058.12402)] for his own equation, and later applied to sonic boom applications by *W.D. Hayes, R.C. Haefeli* and *H.E. Kulsrud* ["Sonic boom propagation in a stratified atmosphere with computer program", Aeronautical Research Associates of Princeton, Technical report, NASA CR-1299 (1969)]. It is an elegant way to locate the position of a shock. Its numerical implementation is almost exact, except for an interpolation of Poisson's solution that can be performed at any order. It is also numerically efficient. As it is exact, a single iteration is sufficient to propagate the information at any distance. It automatically manages waveform distortion, formation of shock waves, and shock wave evolution and merging. The theoretical formulation and the principle of the algorithm are detailed and illustrated by various examples of applications.

MSC:

35Q35 PDEs in connection with fluid mechanics
74J30 Nonlinear waves in solid mechanics
35L67 Shocks and singularities for hyperbolic equations
35L70 Second-order nonlinear hyperbolic equations

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Keywords:

nonlinear waves; shock waves; shock Fitting algorithm; Burgers equation

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