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Efficient path tracking methods. (English) Zbl 1230.65059
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Summary: Path tracking is the fundamental computational tool in homotopy continuation and is therefore key in most algorithms in the emerging field of numerical algebraic geometry. Though the basic notions of predictor-corrector methods have been known for years, there is still much to be considered, particularly in the specialized algebraic setting of solving polynomial systems. In this article, the effects of the choice of predictor method on the performance of a tracker is analyzed, and details for using Runge-Kutta methods in conjunction with adaptive precision are provided. These methods have been implemented in the Bertini software package, and several examples are described.

MSC:

65H20 Global methods, including homotopy approaches to the numerical solution of nonlinear equations
14Q15 Computational aspects of higher-dimensional varieties
65L06 Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
65H04 Numerical computation of roots of polynomial equations

Keywords:

path tracking; homotopy continuation; numerical algebraic geometry; polynomial systems; ordinary differential equations; Euler's method; Runge-Kutta methods; precision; adaptive precision; numerical examples; algorithms; predictor-corrector methods; Bertini software package

Software:

Bertini

Full Text: DOI

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