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**Material growth in thermoelastic continua: theory, algorithmics, and simulation.** (English)

Zbl 1227.74018

Comput. Methods Appl. Mech. Eng. 199, No. 17-20, 979-996 (2010).

Summary: A theoretical formulation of stress-induced volumetric material growth in thermoelastic continua is developed. Material growth is regulated by a three-surface activation criterion and corresponding flow rules. A simple model is proposed based on this formulation and its algorithmic implementation, including a method for solving the underlying differential/algebraic equations for growth, is examined in the context of an implicit finite element method. Selected numerical simulations are presented that showcase the predictive capacity of the model for both soft and hard biomaterials.

**MSC:**

74F05 Thermal effects in solid mechanics

74S05 Finite element methods applied to problems in solid mechanics

Cited in 6 Documents

**Keywords:**

growth; continuum modeling; constitutive modeling; activation surfaces; finite element method

**Software:**

FEAPpv; FEAP

**Full Text:** DOI

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